1. (8 points) What two things do you have to show in order to prove that a time series \( X \) is covariance stationary?

2. (8 points) In terms of the ensemble of realizations, what does it mean to say that \( \hat{\rho}(1) \) is an asymptotically unbiased estimator of \( \rho(1) \)?

3. (8 points) If \( X \) and \( Y \) are two random variables such that \( \text{E}(X) = \text{E}(Y) = 0 \), \( \text{Var}(X) = \text{Var}(Y) = 3 \), and \( \text{Cov}(X,Y) = -2 \), what is \( \text{Var}(X-Y) \)?

4. (10 points) What is \( \text{Var}(\hat{X}) \) if \( X \sim \text{CSTS} \) with autocovariance function \( R \) and spectral density \( f \)? What is this if \( X \sim \text{ARMA}(1,1) \) with \( \alpha = 0.5 \), \( \beta = 0.6 \), and \( \sigma^2 = 1 \) when \( n = 100 \)?

5. (10 points) What are \( \gamma_0, \gamma_1, \ldots, \gamma_4 \) in the \( \text{MA}(\infty) \) representation of the \( \text{ARMA}(1,1) \) process in question 4?

6. (12 points) If \( X \sim \text{MA}(2) \) with \( \beta_1 = 1 \), what is the largest that \( \rho(1) \) can be?

7. (12 points) If you know \( X(1), \ldots, X(n) \) from a \( \text{CSTS} \ X \) and the autocovariances \( R \) of \( X \), how do you find the BLUP \( \hat{X}_{nh} \)? Why is this zero for an \( \text{MA}(q) \) process if \( h > q \)? (Be sure to use the prediction normal equations in your answer.)

8. (12 points) If \( X(1), \ldots, X(n) \) is a realization from a \( \text{CSTS} \ X \) having autocovariance function \( R \), then

\[
\text{Var}(\hat{R}(0)) = \frac{2 \sum_{-\infty}^{\infty} R^2(r)}{n}.
\]

What is this for a realization of length \( N \) from a \( \text{WN}(10) \) process and for a realization of length \( n \) from an \( \text{MA}(1) \) having \( \sigma^2 = 5 \) and \( \beta = 1 \)? (Note that the variances of these two processes are the same). Thus, for the same sample size, which sample has the narrower confidence interval, the \( \text{WN} \) sample or the \( \text{MA} \) sample?

9. (20 points) On the next page are the plots of the population correlogram and partial correlogram for 10 different processes. For each of the processes, tell me whether you think it is an \( \text{MA} \), and \( \text{AR} \), or an \( \text{ARMA} \) process. If an \( \text{AR} \) or \( \text{MA} \), tell me what order it is.

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