1. (10 points) What is the correlation matrix of size 5 for an MA(2) process having $\sigma^2 = 2$, $\beta_1 = .3$, $\beta_2 = .9$?

2. (10 points) Suppose $X(t) = \mu + Y(t)$, $t = 1, \ldots, 100$ where $Y$ is an AR(2) process having $\sigma^2 = 1$, $\alpha(1) = .5$, and $\alpha(2) = .9$. If $X = 37.5$, what is a 95% confidence interval for $\mu$?

3. (10 points) a) Why is it bad that the variance of the periodogram is independent of sample size? b) In terms of the ensemble of realizations, what does it mean to say that an estimator $\hat{\theta}$ is an unbiased estimator of the parameter $\theta$?

4. (10 points) For the spectral density estimator

$$\hat{f}_{x,M}(\omega) = \sum_{v=-\infty}^{\infty} \lambda(v/M)\hat{R}(v)e^{-2\pi iv\omega},$$

one popular lag window generator is due to Parzen and is given by

$$\lambda(u) = \begin{cases} 1 - 6u^2 + 6|u|^3, & |u| \leq 0.5 \\ 2(1 - |u|)^3, & 0.5 \leq u \leq 1 \\ 0 & |u| > 1 \end{cases}$$

For scale parameter $M = 5$, what are the weights applied to the $\hat{R}$'s?

5. (10 points) If $X$ and $Y$ are two uncorrelated, covariance stationary time series with means zero and $\text{Var}(X(t)) = 9$ and $\text{Var}(Y(t)) = 16$. What is the value of $\text{Corr}((X(t) + Y(t)), (X(t) - Y(t)))$?

6. (10 points) For an AR(2), show that $\rho(1)$ and $\alpha_1$ have opposite signs. (Hint: Use the Yule-Walker Equation for $v = 1$ to find a formula for $\rho(1)$ in terms of $\alpha_1$ and $\alpha_2$ and then use the fact that $\alpha_2 > -1$.)

7. (10 points) What are the coefficients $\gamma_1, \ldots, \gamma_4$ of the MA($\infty$) representation of an ARMA(1,1) process having $\alpha = .6$ and $\beta = .8$?

8. (10 points) What happens to a) The width of the frequency transfer function of a moving average smoother having $2K + 1$ terms as $K$ increases? b) The width of the lag window as the parameter $M$ increases? c) The width of the spectral window as the parameter $M$ increases? d) The variance of $X$ as the length of a realization increases? e) The BLUP of an AR($p$) as the number of steps ahead $h$ increases?

9. (20 points) On the next page are the plots of the population correlogram and partial correlogram for 10 different processes. For each of the processes, tell me whether you think it is an MA, and AR, or an ARMA process. If an AR or MA, tell me what order it is.