Makeup Exam #1 - Statistics 211 (Fall 2004)

Name:
Student ID:

- This test consists of 8 numbered pages. Make sure you have all 8 pages. It is your responsibility to inform me if a page is missing!!!
- You have 75 minutes to complete this test.
- You may use the provided crib sheets, all appropriate tables and a calculator.
- If I provide partial results—assume they are correct and use them even if they are not.
- If there is no correct answer or if multiple answers are correct, select the best answer.
- There is no penalty to wrong answers... so guess if you do not know an answer.
- It is your responsibility to look at the overhead or blackboard about every 15 minutes and to incorporate any relevant information into your test.
Chapter 1

[1] Actual blood pressure values for nine randomly selected individuals are:

\[118.6, 127.4, 138.4, 130.0, 113.7, 122.0, 108.3, 131.5, 133.2\]

Find the median.

a) 124.7889
b) 127.4
c) 128.7
d) 122.0
e) The median is not defined for this data set.

[2] Suppose the blood pressure of the first individual in the above example is in reality a lot smaller. How does it affect the median of the reported values? Is the median sensitive to outliers in the data?

a) The median would not change at all. The median is sensitive to outliers
b) The median would not change at all. The median is not sensitive to outliers.
c) The median might change a lot. The median is sensitive to outliers.
d) The median might change a lot. The median is not sensitive to outliers.

[3] The mean on the stat 211 exam is 65 and the maximum score is 80. The standard deviation 10. Suppose all the students are given 4 bonus points. The standard deviation of the newly calculated scores is

a) 10
b) 160
c) 14
d) 25.

[4] Which of the following statements is true?

a) Mean, median, and range are measures of location.
b) Trimmed mean is a measure of variability.
c) Interquartile Range \(Q_3 - Q_1\) is a measure of location.
d) Range and standard deviation are measures of variability.
e) None of the above is true.
[5] One of the main characteristic of the above distribution is that it is:
   a) unimodal
   b) bimodal
   c) multimodal
   d) positively skewed
   e) negatively skewed.

[6] For the above distribution what is the approximate mean of this distribution? If possible, use the scale on the x-axis to determine a number.
   a) 1.0
   b) 0.5
   c) 0.0
   d) 0.2 and 0.8
   e) Cannot be determined without the actual data.

[7] The *Interquartile Range* is defined as $Q_3 - Q_1$. For large data sets the percentage of data contained within this range is:
   a) 25%
   b) 50%
   c) 75%
   d) 100%
   e) None of the above.

**Chapter 2**

[8] When I visit the local library, the probability that someone is reading the current issue of *Sports Illustrated* is .4, the probability that someone is reading *Time* is .3, and the probability that at least one of these two magazines is being read by someone is .6. What is the probability that both magazines are being read?
a) .1
b) .3
c) .58
d) .5
e) None of the above.

[9] The probability that a student will make a mistake on a question is .2. If there are 7 questions, and the probability of getting a mistake is independent of the question, what is the probability that no mistakes are made?
   a) $1 - .2^7$
   b) $1 - .8^7$
   c) $.2^7$
   d) $.8^7$
   e) None of the above.

[10] If $B \subseteq A$ (read B contained in A) then $P(A \cup B)$ is
   a) $P(A)$
   b) $P(A) - P(A \cap B)$
   c) $P(A|B)$
   d) 1.0
   e) None of the above.

[11] If two events are mutually exclusive then they are guaranteed to be independent.
   a) True
   b) False.

[12] A system consists of three components connected in series. The system works only if all three components work. The probability that a given component does not work is $W = .1$. The components are independent of each other. What is the probability the system works?
   a) $1 - P(\bar{W}_1 \cap \bar{W}_2 \cap \bar{W}_3)$
   b) $1 - P(W_1 \cap W_2 \cap W_3)$
   c) $P(\bar{W}_1 \cup \bar{W}_2 \cup \bar{W}_3)$
   d) $P(\bar{W}_1 \cap \bar{W}_2 \cap \bar{W}_3)$
   e) None of the above.

Chapter 3

[13] Let $X$ and $Y$ be two random variables with means 4 and 5 respectively. $E(X + Y)$ is
   a) 9
   b) 10
   c) 20
   d) $\sqrt{40}$
   e) 19.
The probability a student passes Statistics 211 is 90%. The average class size is 80 students. Assuming successive attempts are independent, we want to know how many times a student needs to take Statistics 211 before she passes. The relevant distribution to answer this question is?

a) $X \sim \text{negative binomial}(r = 2, p = .9)$
b) $X \sim \text{geometric}(p = .9)$
c) $X \sim \text{binomial}(n = 80, p = .9)$
d) $X \sim \text{hypergeometric}(N = 80, M = 72, n = 8)$
e) $X \sim \text{poisson}(\lambda = 72)$.

Suppose $X$ follows a negative Binomial distribution with mean 12. $E(3X^2 + 2)$ is

a) 5
b) $3 \times 12^2 + 2$
c) 144
d) 200
e) None of the above.

The average number of accidents at the intersection of Wellborn and HW 2818 is 5.2 per year. What is the probability that exactly 2 accidents occur in a given year?

a) 0.047
b) 0.075
c) 0.109
d) 0.125
e) None of the above.

Suppose that on the average 2 persons in 1000 make a numerical error in preparing their income tax returns. If 4000 forms are selected at random and examined, find the probability that 6, 7, or 8 of the forms will be in error.

a) $P(6 \leq X \leq 8)$ where $X \sim \text{binomial}(n = 1000, p = 0.002)$
b) $P(X = 6) + P(X = 7) + P(X = 8)$ where $X \sim \text{geometric}(p = 0.002)$
c) $1 - P(6 \leq X \leq 8)$ where $X \sim \text{negative binomial}(r = 2, p = 0.002)$
d) $P(6 \leq X \leq 8)$ where $X \sim \text{hypergeometric}(N = 4000, M = 1000, n = 2)$
e) $P(6 \leq X \leq 8)$ where $X \sim \text{poisson}(\lambda = 8)$.

A local television station sells 15, 30 and 60 second advertising spots. Let $x$ denote the length of a randomly selected commercial appearing on this station, and suppose that the probability distribution of $x$ is given in this table:

<table>
<thead>
<tr>
<th>$x$</th>
<th>15</th>
<th>30</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p(x)$</td>
<td>.1</td>
<td>.4</td>
<td>.5</td>
</tr>
</tbody>
</table>

Find the average length of the commercials appearing on this station.

a) $\bar{x} = \frac{1}{n} \sum x = \frac{1}{3}(15 + 30 + 60) = 35$
b) $\bar{x} = \sum x = 15 + 30 + 60 = 105$
c) $\bar{x} = \frac{1}{n} \sum x \cdot p(x) = \frac{1}{3}(15(.1) + 30(.4) + 60(.5))$
d) $\bar{x} = \sum x \cdot p(x) = 15(.1) + 30(.4) + 60(.5)$
e) $\bar{x} = \frac{1}{n} p(x) = \frac{1}{3} (.1 + .4 + .5)$. 
What is $P(X \geq 10)$ where $X \sim \text{Poisson}(\lambda = 10)$?

a) $\sum_{x=10}^{\infty} \frac{e^{-\lambda} \lambda^x}{x!}$

b) $\sum_{x=0}^{10} \frac{e^{-\lambda} \lambda^x}{x!}$

c) $\sum_{x=0}^{\infty} \frac{e^{-\lambda} \lambda^x}{x!}$

d) $0.583$

e) $0.125$.

The average number of field mice per acre in a 7 acre field is estimated to be 5. Find the probability that 2 or fewer field mice are found on any given acre.

a) $P(X \leq 2)$ where $X \sim \text{negative binomial}(r = 5, p = \frac{2}{7})$.

b) $P(X \leq 2)$ where $X \sim \text{binomial}(n = 5, p = \frac{2}{7})$.

c) $0.030$

d) $0.040$

e) $0.125$.

If $X \sim \text{binomial}(n = 20, p = .2)$ then $P(X < 7)$ is

a) $0.780$

b) $0.891$

c) $0.913$

d) $0.982$

e) None of the above.

Suppose $X \sim \text{Geometric}(p)$. To calculate mean we need to solve

a) $\int_0^1 x q^x p dx$

b) $\sum_{x=0}^{\infty} x q^x p$

c) $q \sum_{x=1}^{\infty} (x-1) q^{x-1} p$

d) $\int_0^\infty x q^x p dx$

e) None of the above.

To calculate the mean of a Poisson distribution with parameter $\lambda$, we must compute:

a) $\sum_{i=1}^{\infty} \frac{e^{-\lambda} \lambda^i}{(i-1)!}$

b) $\sum_{i=0}^{\infty} \frac{e^{-\lambda} \lambda^i}{i!}$

c) $\int_0^\infty (x - \mu)^2 \frac{e^{-\lambda x}}{x!} dx$

d) $\int_0^\infty x \frac{e^{-\lambda x}}{x!} dx$

e) None of the above.
Chapter 4

[24] Suppose $X$ has the following pdf.

$$f(x) = \begin{cases} \frac{k}{x^4} & x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

What value of $k$ makes $f(x)$ a legitimate pdf?

a) $1/3$

b) $1$

c) $4$

d) $3$

e) None of the above.

[25] For the following PDF:

$$f(x) = \begin{cases} .75(1 - x^2) & -1 \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute $P(X > 0)$.

a) $\int_{0}^{1} .75(1 - x^2)dx$

b) $\int_{-1}^{0} .75(1 - x^2)dx$

c) $\frac{d}{dx}.75(1 - x^2)$

d) $1.1$

e) None of the above.

[26] For the following PDF in the question above, what is $E[X]$?

a) $\int_{-1}^{1} x \cdot F(x)dx$

b) $\int_{-1}^{1} (.75x - .75x^3)dx$

c) $\int_{-1}^{x} .75(1 - x^2)dx$

d) $\frac{d}{dx}.75(1 - x^2)$

e) None of the above.

[27] How do we calculate the CDF of the PDF in the question above?

a) $\frac{d}{dx}.75(1 - x^2)$

b) $\int_{-1}^{1} .75y(1 - y^2)dy$

c) $\int_{-1}^{x} .75(1 - y^2)dy$

d) We can go from CDF to PDF — not the other way.

e) None of the above.

[28] $X \sim \text{Exponential}(\lambda = 2)$. The CDF of the Exponential distribution is:

$$F(x; 2) = \begin{cases} 1 - e^{-2x} & x \geq 0; \\ 0 & x < 0. \end{cases}$$

Find $P(1 \leq x \leq 2)$ (you can assume the numerical answers are correct).
a) \[ \int_1^2 (1 - e^{-2x}) \, dx \]
b) \[ \int_{-\infty}^{2} (1 - e^{-2x}) \, dx \]
c) \[ F(1; 2) = .864 \]
d) \[ F(2; 2) = .981 \]
e) \[ F(2; 2) - F(1; 2) = 0.117. \]

[29] The CDF of a random variable is

\[ F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-(t-\mu)^2/2} \, dt. \]

The pdf of the random variable is
a) \[ e^{-t^2/2} \]
b) \[ \frac{1}{\sqrt{2\pi}} e^{-(x-\mu)^2/2} \]
c) \[ \frac{1}{\sqrt{x^3}} \]
d) \[ \frac{d}{dx} e^{-(x-\mu)^2/2} \]
e) None of the above.

[30] Suppose \( X \sim Gamma(\alpha = 5, \beta = 2) \). \( E(3X+4) \) is
a) 6
b) 5
c) 18
d) 34
e) 64.

[31] Let \( Z \) be a standard normal variable. \( P(|Z| \leq 2.50) \) is
a) 0.9999
b) 0.9876
c) 1.0000
d) 0.9938
e) None of the above.

[32] Suppose life time of a light bulb follows a gamma distribution with mean 4 hrs and standard deviation \( \sqrt{8} \) hrs. The probability that the bulb will last for at least 4 hours is
a) 0.395
b) 0.801
c) 0.594
d) 0.406
e) 0.264.
1.) b, b, a, d, b
6.) b, b, a, d, a
11.) b, d, a, b, e
16.) b, e, d, a, e
21.) c, b, a, d, a
26.) b, c, e, b, d
31.) b, d