When we use the $z$, the Standard Normal, curve, we are assuming, not only that the data, or at least the sample mean, follows a normal distribution, BUT ALSO the true variance of the data, $\sigma^2$, is known. This never happens in real life, so what can we do? We can estimate the true standard deviation, $\sigma$, with the sample standard deviation, $s$. But there is a problem: 1. we’re now estimating 2 things, AND 2. the sample sd, $s$, can underestimate as often as overestimate. To compensate, we use a $t$ instead of a $z$.

The distribution of $t$ is quite similar to the $z$, the Standard Normal. It is centered at zero, but instead of defining the spread by the standard deviation, $\sigma$, it is defined by the degrees of freedom or just $df$. For the one-sample case, the $df$ of $t = n-1$, the sample size minus 1. [We lose one degree of freedom because now if we know $\bar{x}$, $s$, and $n-1$ of the observations, the last ($n^{th}$) observation is fixed.] As our sample size and hence the $df$ increases, the $t$ distribution gets taller and less spread out. This equates to $s$ getting closer to the true value, $\sigma$, as we get more data. When $s=\sigma$, the $t$ confidence interval will be wider than the $z$ since we are unsure of the true the variability of the data. There are times when our estimate, $s$, is so much less than $\sigma$ that even using the $t$ doesn’t quite give us a wide enough interval, but this is rare. What happens when our interval is too narrower? Well, we may not cover the true mean!