Homework 4

Problems 1 - 5: numbers 2.8 (a), 2.11 (b), 2.14 (a), 2.16 and 2.17 from the textbook.

Problem 6: Let X be a random variable with range $\{0, 1, 2, \ldots\}$. The discrete version of the formula from problem 2.14 in the textbook, i.e. 2.14 (b), can be written

$$EX = \sum_{k=1}^{\infty} P(X \geq k).$$

(The proof is analogous to that of 2.14 (a).) Use this formula to solve the following problem.

A fair dice is thrown n times. The sample space is $S = \{1, 2, \ldots, 6\}^n$, the outcomes are of the form $s = (s_1, \ldots, s_n) \in S$. Let Y_n denote the largest of the results thrown, i.e. Y_n is a r.v. with $Y_n(s_1, \ldots, s_n) = \max_{1 \leq k \leq n} s_k$.

a) Find EY_n and show

$$\lim_{n \to \infty} E(Y_n) = 6.$$

b) Show

$$\lim_{n \to \infty} Var(Y_n) = 0.$$

Problem 7: Let X be a discrete random variable that takes on values $0, 1, 2$ with probability 1/2, 3/8, 1/8, respectively.

a) Find $E(X)$.

b) Find the pmf of $Y = X^2$ and use it to find $E(Y)$.

c) Use the definition of $E\{g(X)\}$, where $g(X)$ is a function of X, to find $E(X^2)$ and compare to your answer in part (b).

d) Find $Var(X)$.