Homework 3

Problem 1.

a) Consider $f(x) = cx^{-3}1_{(1,\infty)}(x)$ with $c \in \mathbb{R}$. Determine c such that f is a probability density function and find the corresponding cumulative distribution function F.

b) Consider $f(x) = c\sin(x)1_{(0,\pi)}(x)$ with $c \in \mathbb{R}$. Determine c such that f is a probability density function and find the corresponding cumulative distribution function F.

Problem 2. Consider a distribution P given by a pdf $f(\cdot)$ with $f(x) = 0$ for all $x \leq 0$. Assume F is a continuous antiderivative of f with $F'(x) = f(x)$ for all $x > 0$ and $F(x) = 0$ for all $x \leq 0$. Then, by definition,

$$P((a,b]) = \int_a^b f(x)\,dx = F(b) - F(a) \quad \text{for } 0 \leq a \leq b < \infty.$$

Further assume that there is a constant $\lambda > 0$ with

$$(*) \qquad \lim_{0<h\to 0} \frac{P((t,t+h]\,|\,(t,\infty))}{h} = \frac{1}{\lambda} \quad \text{for all } t > 0.$$

a) Show that $f(x) = \frac{1}{\lambda}e^{-x/\lambda}$ for all $x > 0$ (exponential distribution).

b) If P is the distribution of the lifetime of an individual or object, then $P((t,\infty))$ is the probability that the individual survives at least t time units. Given this, how would you interpret equation $(*)$? Is it reasonable to assume that humans' lifetimes are exponentially distributed?

Problem 3. Consider a random variable X with a Poisson(1) distribution and the transformation $Y = (X+1)^{-1}$. Specify the probability mass function $f_Y(y)$ of Y. Check your answer by verifying that $f_Y(y)$ satisfies the criteria for a probability mass function.

Problems 4 - 6: numbers 1.53, 1.55, 2.1 from the textbook.