Homework 2

Problems 1-3: numbers 1.36, 1.39 and 1.44 from the textbook.

Problem 4. Suppose there were seven road accidents in one week. What is the probability that they all happened on different days?

Problem 5. Consider

$$\mathcal{S} = \{1,2,3,4\};\ P = \text{discrete uniform distribution on } \mathcal{S};$$
$$A = \{1,2\},\ B = \{1,3\},\ C = \{1,4\}.$$

Are the events A, B and C independent? If yes, in which sense? Explain your answer.

Problem 6. Consider the probability mass function of the binomial distribution

$$f(j) = b_j(n,p) = \binom{n}{j} p^j (1-p)^{n-j}, \quad j = 0,1,\dots,n,\ n \in \mathbb{N},\ 0 < p < 1.$$

Show that the binomial distribution can sometimes be approximated by the Poisson distribution. Formally, for $p = p_n \in (0,1)$, $\lim_{n\to\infty} p_n = 0$, $\lim_{n\to\infty} np_n = \lambda > 0$ holds

$$\lim_{n\to\infty} b_j(n,p_n) = e^{-\lambda}\frac{\lambda^j}{j!} \quad \text{for every } j \in \{0,1,\dots\}.$$

Hint: $\lim_{n\to\infty}\left(1 + \frac{\lambda+\varepsilon_n}{n}\right)^n = e^{\lambda}$ for $\lambda \in \mathbb{R}$, $\lim_{n\to\infty}\varepsilon_n = 0$.

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Please label each page of your homework clearly with your name IN BLOCK CAPITALS and your UIN. If you use more than one sheet of paper, please staple the sheets together.