1. Bayes Estimation for the Pareto Distribution
   (a) Problem 1 (c), p. 299, Shao.
   (b) Problem 2 (c), p. 299, Shao.
   (c) Problem 29 (c), p. 303, Shao.

2. Bayes Estimation for the Binomial Distribution
   (a) Problem 19 (b), p. 302, Shao.
   (b) Problem 30, p. 303, Shao.

3. Let \((X_1, X_2, X_3)\) have a multinomial distribution based on \(n\) trials with cell probabilities given by
   \[
   \pi_1(\theta) = \frac{1}{3} - \theta, \quad \pi_2(\theta) = \frac{2}{3} - \theta, \quad \pi_3(\theta) = 2\theta, \quad \text{where } 0 < \theta < \frac{1}{3}.
   \]
   (a) Obtain the limiting distribution of the maximum likelihood estimator and hence, of the minimum chi-squared and modified minimum chi-squared estimators.
   (b) Suppose that \(n = 100, \ x_1 = 20, \ x_2 = 50, \ x_3 = 30\). Compute the following estimates:
      (i) maximum likelihood estimate
      (ii) minimum chi-squared estimate
      (iii) minimum modified chi-squared estimate

4. Let \(X_1, \ldots, X_n\) be a random sample from a continuous population with c.d.f. \(F(x - \theta)\). Consider the estimator \(\theta^*\) of \(\theta\) which is obtained by solving the equation
   \[
   \sum_{i=1}^{n} \{\Phi(X_i - \theta) - \frac{1}{2}\} = 0,
   \]
   where \(\Phi\) is the standard normal c.d.f.
   (a.) Obtain the influence function of the estimator and draw a rough sketch of it. Discuss whether or not the estimator is robust.
   (b.) Assume that \(F = \Phi\). Show that the asymptotic variance is \(1/[12(\text{E}[\phi(X - \theta_0)])^2]\), where \(\phi\) is the standard normal p.d.f. Show that the asymptotic efficiency of the estimator \(\theta^*\) is \(3/\pi\).

5. Let \(X_1, \ldots, X_n\) be a random sample from a continuous population with c.d.f. \(F(x)\). The variance functional is
   \[
   \gamma = T(F) = \int_{-\infty}^{\infty} t^2 dF(t) - (\int_{-\infty}^{\infty} t dF(t))^2.
   \]
(a.) Express the estimator \( \hat{\gamma} = T(F_n) \) in terms of \( X_1, \ldots, X_n \). Here \( F_n \) is the empirical c.d.f.

(b.) Derive the influence curve of this functional and draw a typical graph.

(c.) What are the gross error sensitivity \( \gamma^*(T, F) = \sup_{-\infty < x < \infty} |IC_{T,F}(x)| \) and the local shift sensitivity \( \lambda^* = \sup_{-\infty < x \neq y < \infty} |(IC_{T,F}(x) - IC_{T,F}(y))/(x - y)| \)?

6. In a study of stellar kinetics, the velocity orthogonal to the galactic plane of RR Lyrae variable stars was measured. The data in the following table are given for three locations of stars:

<table>
<thead>
<tr>
<th>Stars</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disk Stars</td>
<td>4,44,-23,-32,26,13,34,-24,-10,-34,72,-26,-32,-144,3,0,-43</td>
</tr>
<tr>
<td>Intermediate</td>
<td>0,45,6,-48,65,55,-69,-77,117,33,-17,64,5,-286,-175,-29,-63,</td>
</tr>
<tr>
<td>Halo Stars</td>
<td>23,58,69,7,-1,25,-268,-44,87,-102,-42,25,16,62,31,5,21,-77,-63</td>
</tr>
<tr>
<td></td>
<td>214,129,34,-31,155,76,-18,-96,33,-81,-6,20,95,-72,110,90,-118,-61,-4</td>
</tr>
</tbody>
</table>

A plausible model for these data is to assume that \( X_1, \ldots, X_n \) are i.i.d. \( F(x/\sigma) \), where \( \sigma \) can vary for the three populations of stars.

(a.) Suppose that \( F = \Phi \). It is often reasonable to treat scale estimators on the log scale. Let \( \hat{\sigma} \) be the m.l.e. of \( \sigma \). Obtain the asymptotic distribution of \( \hat{\sigma} \) and \( \log(\hat{\sigma}) \). Also, for each of the 3 data sets, obtain estimates of \( \sigma \), \( \log(\sigma) \), and the standard errors of both estimators.

(b.) Suppose now the \( F = F_0 \). We can use the influence function to obtain a nonparametric estimate of the standard error of the scale estimate. Use Problem 5.(b.) to show that

\[
\sqrt{n}(\hat{\sigma}^2 - \sigma^2(F_0)) \xrightarrow{d} N(0, E_{F_0}[(X - \mu(F_0))^4] - \sigma^4(F_0)).
\]

Apply the \( \delta \)-method to show that to obtain the asymptotic distribution of \( \log(\hat{\sigma}) \). Use this to motivate the following estimate (instead of 0.5) of the standard error of \( \log(\hat{\sigma}) \):

\[
\text{Estimated std. error} = \left[ \frac{1}{4n\hat{\sigma}^4} \left\{ \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^4 - \hat{\sigma}^4 \right\} \right]^{1/2}.
\]

Obtain the nonparametric estimated standard errors for \( \hat{\sigma} \) and \( \log(\hat{\sigma}) \).

(c.) Use the bootstrap to estimate the bias and standard errors for the estimates for \( \sigma \) and \( \log(\sigma) \) used in part (a.). Compare these estimates to the theoretic values found above.

(d.) Use the jackknife to estimate the bias and standard errors for the estimates for \( \sigma \) and \( \log(\sigma) \) used in part (a.). Compare these estimates to the theoretic values found above and to those found using the bootstrap.

7. Problem 90, p. 390, Shao.