Finding the value on the \textit{x-axis}

\begin{enumerate}
\item Suppose $X \sim N(5,4)$

\begin{itemize}
\item Question: What is the value of $x$, such that
\end{itemize}

\begin{itemize}
\item Areas under both graphs (probabilities) $= 0.71$
\end{itemize}

\begin{align*}
&\Rightarrow P\left( \frac{X-5}{\sqrt{4}} \leq z \right) = 0.71 \\
&\text{Look inside normal tables for 0.71.}
\end{align*}

We see that $P(Z \leq 0.55) = 0.71$. 

\end{enumerate}
Therefore

\[ \frac{x - 5}{\sqrt{4}} = 0.55 \Rightarrow x = 6.1 \]

The answer is 6.1. That is, if \( X \sim N(5, 4) \), then
\[ P(X \leq 6.1) = 0.71 \]

**Suppose** \( X \sim N(10, 9) \)

Find the \( x \)-value such that \( P(X \geq x) = 0.35 \)?
Answer: From the density plot we see that it is equivalent to finding the $x$ such that $P(X \leq x) = 0.65$

Both probabilities (areas) are the same.

$$\frac{x-10}{\sqrt{\sigma^2}} = 0.39$$

$$P(X \leq x) = P\left(\frac{X-10}{\sqrt{\sigma^2}} \leq \frac{x-10}{\sqrt{\sigma^2}}\right) = P\left(Z \leq \frac{x-10}{\sqrt{\sigma^2}}\right) = 0.65$$

Look 'inside' the normal tables to find $P(Z \leq 0.39) \approx 0.65$.

Solving for $\frac{x-10}{\sqrt{\sigma^2}} = 0.39$ gives $x = 11.17$.

$p(x > 11.17) = 0.35$
Suppose \( Z \sim N(0, 1) \) [standard normal].

Find \( x \), such that \( P(-x \leq Z \leq x) = 0.95 \)

We can see that \( P(-x \leq Z \leq x) = P(Z \leq x) - P(Z < -x) \).

Let us look carefully at what \( P(Z < -x) \) means:

Clearly from the picture above \( P(Z < -x) = 1 - P(Z > -x) \)

But \( \theta \) by symmetry \( P(Z > -x) = P(Z < x) \)

\(*\) This is because the standard normal is symmetric about zero.
Therefore

\[ P(z < -x) = 1 - P(z \geq -x) = 1 - P(z \leq x). \]

Thus means that:

\[ P(-x \leq z \leq x) = P(z \leq x) - P(z \leq -x) \]

\[ = P(z \leq x) - [1 - P(z \leq x)] \]

\[ = 2P(z \leq x) - 1. \]

Finally (!) we can solve the equation:

\[ P(-x \leq z \leq x) = 2P(z \leq x) - 1 = 0.95 \]

This means that

\[ P(z \leq x) = \frac{0.95}{2} = 0.975. \]

Look inside tables

This gives

\[ P(z \leq x) = 0.975 \]

\[ x = 1.96 \]

\[ -x = -1.96 \]

\[ 0.95 \]
Alternative Solution to $P(-x \leq z \leq x) = 0.95$.

By symmetry the area $P(-x \leq z \leq 0) = \frac{0.95}{2}$.

This area = $\frac{1}{2} - 0.475 = 0.025$.

Because the area before 0 is $\frac{1}{2}$.

$P(z \leq -x) = 0.025$, now look up normal tables to give $x = 1.96$. 