The measure of spread and its influence on the sample mean

Example 1: The density of a 'large spread population'.

The sample average $\bar{x}$

The crosses denote the observations of a 'typical' sample. You see that the probability of observing these points is not very small. So the probability of observing this sample is not small. I.e., a good chance of observing it.

You see that the sample average $\bar{x}$ is quite far from the population mean $\mu$. That is, the distance $(\bar{x} - \mu)$ is large.
Because there is a large amount of spread (variation) in the population, we are likely to observe a sample mean \( \bar{x} \) that is far away from the population mean \( \mu \).

**Example 2:** The density of 'small spread population'.

Unlikely to observe anything here (probability very small-chance small).

The crosses denote a 'typical' sample. See that they are 'likely' to be close to the population mean. Therefore the sample mean is likely to be close to the population mean. The distance \( \mu - \bar{x} \) is likely to be small.
The Variance (sample variance)

The sample $\bar{x}$

Each of lines above $A, B, C, D, E, F$ denote the distance between the observation and the center $\bar{x}$. The variance is the average squared of these distances.

The variance is measuring the average spread from the center.

$$\text{variance} \approx \frac{1}{6} \left[ A^2 + B^2 + C^2 + D^2 + E^2 + F^2 \right]$$
Example 1 (large variance).

The average square distance will be quite large, because of the spread in the data.

- The variance does not depend on how large the values $X_1, X_2, X_3, \ldots, X_7$ are, it depends on how spread out they are.

Example 2 (small variance).
The observations are not spread out much. The average squared distance is small. Therefore the variance is small.

I have illustrated the variance for the sample variance.

The same ideas hold for the population variance, just in this case the 'average squared distance' is now a weighted square distance between each point and the population mean. The weight happens to be the density function - but you do not need to know this.