STAT301 Homework 5

(1) Here we match probabilities to t-transforms.

Using the t-tables find the most accurate probability bounds for the following t-transforms. For example, if I ask for the area to the left of \( t = 2 \) for a t-distribution with 6 degrees of freedom the best bound is that the probability will be between 95% and 97.5%. Making a plot will help.

(a) The area to the right of \( t = 2.3 \) for a t-distribution with 20 degrees of freedom.
(b) The area to the left of \( t = 0.6 \) for a t-distribution with 10 degrees of freedom.
(c) The area to the right of 0.9 for 11 degrees of freedom.
(d) The area to the right of \(-0.9\) for 11 degrees of freedom.

(2) More determining the hypothesis practice.

Make sure to identify the parameters you need to use to state the hypotheses.

(a) A sociologist asks a large sample of students which television channel they liked best. She suspects a higher percentage of males than females will name MTV as their favourite tv channel.
(b) Last year, online technicians took an average of 0.4 hours to respond to IT trouble calls. Do this years data show a different average response.

(3) All the below tests are done at the 5% level.

I am conducting a test on female heights. I take a sample of 10 women, the average based on these 10 women is 67 inches and the sample standard deviation is \( s = 2 \) (use the t-distribution with 9df instead of the normal).

(a) I test \( H_0 : \mu = 66.5 \) against the alternative \( H_A : \mu \neq 66.5 \). What is the p-value and the conclusion of the test.
(b) I test \( H_0 : \mu = 66 \) against the alternative \( H_A : \mu \neq 66 \). What is the p-value and the conclusion of the test.
(c) Based on the above what are you conclusions about accepting the null in a hypothesis test.

(4) In ancient China it seems that owning a pig was a symbol of wealth. Evidence comes from examining burial sites and comparing artifacts in burial sites with and without pig skulls. A study of burials from around 3500 BC concluded that there are ‘striking differences in grave goods between burials with pigs skulls and burials without them....A test indicates that the two samples of total artifacts are statistically different at the 0.1% level’. Explain ‘why statistically different at the 0.1% level’ gives good reason to suppose there really is a systematic difference between burials that contain pig skulls and those that lack them.

Hint: The null hypothesis is that there is no difference in wealth between those buried with a pig head and those not buried with a pig head, the alternative hypothesis is that those buried with a pig head tend to be wealthier.
(5) A 99% confidence interval for a population mean is [86, 100]. The tests below are done at the 1% level.

(a) Can you reject the hypothesis that $H_0 : \mu = 90$ for the alternative $H_A : \mu \neq 90$. Why?

(b) Can you reject the $H_0 : \mu = 105$ for the alternative $H_A : \mu \neq 105$? Why?

(6) In this question we see how sample size influences the p-value.

You are testing the null hypothesis $H_0 : \mu = 0$ against the alternative $H_A : \mu > 0$ using the significance level 5%. Assume the standard deviation is known with $\sigma = 14$ (use normal distribution).

(a) The sample mean based on a sample of size 10 is $\bar{x} = 4$. Find the z-transform and corresponding p-value. Can you reject the null at the 5% level.

(b) The sample mean based on a sample of size 20 is $\bar{x} = 4$. Find the z-transform and corresponding p-value. Can you reject the null at the 5% level.

(c) The sample mean based on a sample of size 40 is $\bar{x} = 4$. Find the z-transform and corresponding p-value. Can you reject the null at the 5% level.

(d) What does this example tell us about the ability of the test to reject the null as the sample size increases?

(7) In this question we consider the ability of a test to reject the null for an important alternative. This chance is called the power of the test. We worked through a similar example in class.

Patients with chronic kidney failure may be treated by dialysis using a machine that removes toxic waste from the blood, a function normally performed by the kidneys. However, kidney dialysis can cause other problems such as phosphorus retention. A healthy mean phosphorus levels should be 4.8 $\text{mg/dl}$ or less. In order to diagnose a problem, 6 blood samples are taken. It is assumed the blood samples vary according to a normal distribution with standard deviation $\sigma = 0.9 \text{mg/dl}$.

(a) Explain why the null and hypothesis to detect a mean phosphorus level over 4.8 $\text{mg/dl}$ is $H_0 : \mu = 4.8$ against $H_A : \mu > 4.8$?

(b) What is the standard error for the sample mean based on a sample of size 6?

(c) The test are done at the 1% level.

Doctors want an automatic way to do the test, where given the average based on 6 blood samples they compare it to a value and decide whether there is evidence to reject the null and conclude that the patient has a high blood level. What threshold would you advice doctors to use to reject the null and conclude the patient has high phosphorus levels.

(d) Based on the test above, if 1000 patients have normal phosphorus levels and are tested using the above procedure, how many will be (wrongly) diagnosed with having a high mean phosphorus level?
(e) Doctors tell you that a mean phosphorus level that is equal to is over 5.5 is considered extremely dangerous. Using the same calculation we did in class (and can be found in the slides) calculate the chance the above test will be able to detect patients with extremely high phosphorus levels.

This is the probability of rejecting the test and determining $\mu > 4.8$ when the patient is seriously ill. It is called the power of the test.

(f) Describe what a type I error is and what is it for the above test?

(g) Describe what a type II error is and what is it for the above test (when we want to detect the case $\mu \geq 5.5$)?

(h) One patient has 6 blood tests taken. These are the patient’s results

5.4, 5.2, 4.5, 4.9, 5.7, 6.3.

Is there evidence to suggest that the mean level is over 4.8 (use in this calculation that the standard deviation is $\sigma = 0.9$, which is known)?