STAT301 Homework 3

In this homework we consider how to design an experiment correctly (without bias), evaluating normal probabilities and applying the central limit theorem.

(1) A teacher asks her class "How many children are there in your family, including yourself"? The mean response for her class is 3 children per family.

According to the U.S. Census Bureau, in 2012 the number of children per household is 1.86 children. Why is the teacher’s sample biased towards higher outcomes?

(2) What is wrong with the following randomization procedures? How would you improve them?

(a) Twenty students are to be used to evaluate a new treatment. Ten men are assigned to receive the treatment whereas ten women are assigned to be controls.

(b) Ten subjects are assigned to two treatments, supposedly 5 subjects to each treatment. For each of the ten subjects a coin is tossed. If the coin comes up heads the subject is assigned to the first treatment, if the coin comes up tails the subject is assigned to the second treatment.

(3) A patient is classified as having gestational diabetes if the glucose level is above 140 miligrams per deciliter one hour after ingesting a sugary drink. Lucy’s measured sugar level varies according to a normal distribution with mean $\mu = 125\text{ mg/dl}$ and standard deviation $10\text{ mg/dl}$.

Since the her mean level is below 140 mg/dl she does not have gestational diabetes. However, in reality the mean level is unknown, all that is known are readings taken from blood samples. Therefore, below we want to evaluate the chance of wrongly diagnosing gestational diabetes based on the samples taken.

(a) Suppose one single measurement is made (one blood sample), what is the probability that she will be misdiagnosed as having gestational diabetes (in other words what is the chance that her measurement will be above 140 mg/dl given that a single measurement is normally distributed with mean $\mu = 125\text{ mg/dl}$ and standard deviation $10\text{ mg/dl}$).

(b) Instead suppose that on three separate days measurements are made and the average measurement is taken over these three days. What is the probability that she will be misdiagnosed as having gestational diabetes (in other words what is the chance that her average over these three measurements will be above 140 mg/dl)?

*Hint: What is the distribution of the sample mean based on three measurements given that a single measurement is normally distributed with mean $\mu = 125\text{ mg/dl}$ and standard deviation $10\text{ mg/dl}$?*

(c) Compare your solutions from part (a) and part (b). What have you notice about the probability of false diagnosis as a larger sample is used?
(4) Suppose the scores of high school ACT test have mean 19.2 and standard deviation 5.1. As we discussed in class, ACT scores are only very approximately normally distributed.

(a) Using the normal distribution, what is the approximate probability that a single randomly selected student will score 23 or higher?

(b) A simple random sample of 25 students is taken. What is the mean and standard deviation of the average score (sample mean $\bar{x}$) of these 25 students?

(c) Using the normal distribution, what is the approximate probability that the sample mean score of these 25 randomly selected students will be 23 or higher?

(d) Which of your Normal probability calculations (a) and (c) will be the most accurate, give a reason for your answer?

(5) The weight of airline passengers including carry-on luggage vary from passenger to passenger and airlines must take into account the passengers weight in order to determine the distribution weight in their aircraft.

The mean weight of an airline passengar including carry-on luggage has a mean of 190 pounds and standard deviation 40 pounds (however, there is no real guarantee it is normally distributed). The FAA stipulates that a plane carrying 30 passengers cannot take a passenger load which is greater than 6000 pounds. Using the information in question to calculate the probability that 30 passengers (with carry-on luggage) will exceed this weight.

*Hint: If the weight cannot exceed 6000 pounds, what does this tell this mean about the average weight of the 30 passengers? Use this information together with the central limit theorem to calculate the chance.*