Discussion on

Statistical Methods for Multivariate Spatial and Spatial-Temporal Processes

Mikyoung Jun
mjun@stat.tamu.edu

Texas A&M University

August 1, 2010
Linear Model of Coregionalization (LMC)

- Easy to build the covariance model for any number of processes, do not need to worry about positive definiteness
- Extension for nonstationary processes:
  - Suppose we model temperature and precipitation jointly.
    \[
    T(s) = \{a_1 \cdot A(s) + b_1 \cdot L(s)\}Z_1(s) + \{a_2 \cdot A(s) + b_2 \cdot L(s)\}Z_2(s),
    \]
    - Interpretation of the coefficients $a_i$ and $b_i$’s are tricky
- We could let the coefficients of the independent processes be random but the implementation could be complex and there may be problems with identifiability or overparametrization
Matérn cross-covariance function

- Easy to implement, especially parsimonious version
- Each covariance parameter is assigned to each marginal process or cross-covariance for a pair of processes: easy to interpret the parameters
- In my limited experience, this model fits better (gives higher loglikelihood values) compared to the isotropic version of the LMC models with comparable number of covariance parameters
Matérn cross-covariance function

Extension to nonstationary version?

Suppose $Z_1$ and $Z_2$ are bivariate processes with proposed Matérn cross-covariance structure

If we let $W_i(x) = a_i(x)Z_i(x)$, resulting covariance structure for $W_1$ and $W_2$ can be nonstationary. But the cross-correlation structure is still isotropic

How can we achieve nonstationary (cross-) correlation structure?

Currently cross correlation function is a constant multiplied by a Matérn function, and thus monotonic function of distance

Could we let the co-located correlation coefficient parameter vary over space and achieve similar model classes?
Cross-covariance model via latent dimensions

- Convenient to create rich classes of cross-covariance models

- Same idea can be applied to univariate nonstationary covariance model to achieve nonstationarity in space (and time)

- In that case, what should we do with the dimension for cross-covariance component?
Non-parametric cross covariograms

- No need to estimate covariance parameters, assume certain parametric structure
- Computationally fast and efficient
- About finding the pairs of $Z$ values that are perfectly correlated:
  - How sensitive is the result to this estimation?
- How can we improve the poor estimation around the origin when the number of eigenterms is small?
Processes on a sphere?

- Nonstationarity with respect to latitude is essential, asymmetry, nonseparability, and other complex structures may be more serious.

- Jun (2009):
  - Nonstationary covariance model for multivariate process on a sphere:
    \[
    Z_i(L, l) = \sum_{k=1}^{m} \left\{ a_{ik}(L) \frac{\partial}{\partial L} + b_{ik}(L) \frac{\partial}{\partial l} \right\} Z_{0k}(L, l)
    \]
  - Differential operators are defined in $L_2$ sense
  - With small number of covariance parameters, we get the covariance dependence on latitude
Other issues with cross-covariance models

- How much does it matter to have cross-covariance structure (rather than assuming independence) in prediction?

- Suppose marginal covariance has some nice properties such as stationarity or isotropy.
  - Does this require that the cross covariance structure should have the same nice property?