I. (20 points) **Instructions.** In the following two descriptions of experiments, provide the following information:

1. Type of Randomization, for example, CRD, RCBD, LSD, BIBD, SPLIT-PLOT, SUBSAMPLING, etc.;
2. Type of Treatment Structure, for example, Single Factor, Crossed, Nested, etc.;
3. Identify each of the factors as being Fixed or Random;
4. Describe the Experimental Units and/or Measurement Units.
5. Identify any Covariates:
6. An ANOVA Table Including: Sources of Variation and Degrees of Freedom

A. An industrial engineer is investigating the difference of four types of assembly methods relative to the assembly time for a color television component. The engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method used in the assembly process. Therefore, four sequences of applying the assembly methods were selected for the study such that each assembly method appeared first, second, third, or fourth in one of the sequences. Sixteen operators are randomly selected for the study with four operators randomly assigned to each of the four sequences. Thus, each operator is observed applying all four assembly methods.

1. Type of Randomization:
2. Type of Treatment Structure:
3. Identify each of the factors as being Fixed or Random:
4. Describe the Experimental Units and/or Measurement Units:
5. Identify any Covariates:
6. An ANOVA Table

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. A seafood specialist investigated bacterial growth in oysters subjected to 5 lengths of cold storage times and at 3 storage temperatures. Nine cold storage units were available; three were randomly selected and set at each temperature. Three batches of oysters were obtained from different sources. From each batch, 15 packages were formed and an initial bacterial count was recorded for the 45 packages due to possible differences in bacterial contamination of the packages. Five of these packages were randomly assigned to one cold storage unit of each temperature. At the end of each designated length of storage, one package was randomly selected from each storage unit and a bacterial count taken.

1. Type of Randomization:
2. Type of Treatment Structure:
3. Identify each of the factors as being Fixed or Random:
4. Describe the Experimental Units and/or Measurement Units:
5. Identify any Covariates:
II. (30 pts.) A plant scientist is studying the effect of irrigation and fertilization on the water retention of leaves on various varieties of tobacco plants. She uses two levels of Irrigation (Low and High), three Varieties of tobacco plants (V1, V2, and V3), and three amounts of Fertilizer (10, 20, and 30). Each of the 6 combinations of I and V was randomly assigned to 3 similar plots of land. Each of the 18 plots were then divided into 3 regions and a level of F was randomly assigned to each of the 3 regions. After 10 weeks, a water retention value was determined for each region. The plant scientist wishes to make inferences about only the levels of the three factors I, V, and F used in the experiment.

A. Complete the following AOV table. TH

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
<th>MS</th>
<th>EMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>350</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I*V</td>
<td>200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rep(I,V)</td>
<td>520</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>550</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I*F</td>
<td>250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V*F</td>
<td>150</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I<em>V</em>F</td>
<td>500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ERROR</td>
<td>150</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. At the α = 0.05 level, evaluate the effect of Irrigation, Variety, and Amount of Fertilizer on the mean water retention value.

C. Using the numeric values of the MS's given above and your EMS's, find the values of the following quantities:

1. An estimate of the standard error of the estimated mean water retention value for the High level of Irrigation. What are the degrees of freedom of your estimator?
2. The estimate of the standard error of the estimated difference in mean water retention for the two levels of Irrigation. What are the degrees of freedom of your estimator?
3. The estimate of the standard error of the estimated difference in the mean water retention of two levels of Fertilizer at the Low level of Irrigation. What are the degrees of freedom of your estimator?

III. (10 pts) A researcher designs a factorial experiment with Factor $F_1$ having 3 levels and Factor $F_2$ having 2 levels. The number of replications are $r_{11} = 2, r_{12} = 3, r_{21} = 2, r_{22} = 3, r_{31} = 2, r_{32} = 2$ for the six treatments.

1. The cell means model for this experiment can be expressed as $Y = X\beta + e$. Display the $X$ matrix and $\beta$ vector explicitly for this situation.

2. The effects model $Y_{ijk} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij} + e_{ijk}$ for this experiment, with the constraints used with most computer packages, is equivalent to a regression model. Define the independent variables in the regression model and express as $Y = X\beta + e$. Display the $X$ matrix and $\beta$ vector explicitly for this situation.

IV. (40 pts) For each of the following statements, decide if the statement is true or false and place a T or F to the left of each statement. If the statement is false, make changes in the LAST SENTENCE in the statement in order to make the statement true.

(1) In a Crossover Design, both the sequence effect and the carryover effect were highly significant ($p$-value < .0001). Using the observed data, the significance of the treatment effect can not be determined because it is confounded with the carryover effect.

(2) A covariate was measured along with the responses within a completely randomized design. The researcher determines that the slopes of the 5 treatment lines are different. A comparison of the 5 treatments cannot be conducted because there is an interaction between the covariate and treatment which violates the conditions for analysis of covariance.

(3) A three factor experiment is run with Factor A-fixed, Factor B-fixed nested within Factor A, Factor C-fixed crossed with Factor A. The interaction between factors A and C was not significant and the interaction between factors B(A) and C was not significant. A comparison of the differences in the levels of Factor B can be done using Tukey’s HSD on the means for the levels of Factor B averaged over the levels of Factors A and C.

(4) A $2^{6-2}$ design has resolution V. All four-way or higher order interactions are determined to be essentially zero. Using the data from this experiment, all main effects, two-way interactions, and three-way interactions can be uniquely estimated.

(5) In a split-plot analysis of a repeated measures design, we consider the 5 times at which the measurements are taken to be the split-plot factor. The analysis yields only an approximation to the correct p-values because there is a strong correlation between the repeated measurements taken on the same experimental unit.

(6) A researcher uses a Balanced Incomplete Block Design in her study. In this type of design, every pair of treatments appear in every block.

(7) A researcher is studying the absorption times of two types of antibiotic capsules: $T_1$, $T_2$. There are three capsule wall thicknesses of interest to the researcher: .2, .4, .6 mm. The researcher randomly selects a wall thickness and type of capsule places this setting on the capsule formulation (FORM) machine, and produces a capsule. The researcher then determines the absorption time for the capsule. A pair of contrasts which test the null hypothesis of no interaction in terms of the cell means: $\mu_{11}, \mu_{12}, \mu_{13}, \mu_{21}, \mu_{22}, \mu_{23}$ is $L_1 = \mu_{11} + \mu_{12} + \mu_{13} - \mu_{21} - \mu_{22} - \mu_{23}$ and $L_2 = \mu_{11} + \mu_{12} - 2\mu_{13} + \mu_{21} + \mu_{22} - 2\mu_{23}$

(8) A testing laboratory is evaluating the differences in accuracy of field test devices for detecting $e. coli$ in meat products. From historical data, the value of $\sigma$, the overall measurement error of the devices is taken to be 2 ppm. There are over 250 devices available for detecting $e. coli$ and the testing laboratory decides to randomly select a number of these devices for evaluation. For each of the selected devices, 6 independent determinations of the amount of $e. coli$ in a meat sample will be obtained. The laboratory must select at least 9 testing devices for evaluation in order to obtain an $\alpha=.01$ test having power of at least 0.80 of detecting that the standard deviation in the field test devices is 2.5 ppm.
May 7, 2003

I. (30 points) Instructions. In the following two descriptions of experiments, provide the following information:

1. Type of Randomization, for example, CRD, RCBD, LSD, BIBD, SPLIT-PLOT, etc.;
2. Type of Treatment Structure, for example, Single Factor, Crossed, Nested, etc.;
3. Identify each of the factors as being Fixed or Random;
4. Describe the experimental units and/or measurement units.
5. Identify any covariates:
6. An ANOVA Table Including: Sources of Variation and Degrees of Freedom

A. A flour miller wanted to study the effect that four sizes of roller gaps (.02 cm, .04 cm, .06 cm, .08 cm) have on a flour mill as to the amount of flour produced from five varieties of wheat ($V_1, V_2, V_3, V_4, V_5$). The process is to take a batch of wheat from one variety and run a portion of that batch of wheat with each of the roll gaps. The order of the roll gaps was randomized for each batch of wheat from the selected varieties and the order in which the varieties were run was randomized. Twenty runs could be accomplished during one day. The researcher therefore was able to observe all five varieties under each of the four roller gaps during each day in the study. The whole process was repeated on three days. The moisture content of the wheat being milled can have an affect on the flour production, so the moisture content of each of the portions of wheat from a given variety was measured prior to starting the milling of the wheat.

1. Type of Randomization:
2. Type of Treatment Structure:
3. Identify each of the factors as being Fixed or Random:
4. Describe the experimental units and/or measurement units:
5. Identify any covariates:
6. An ANOVA Table

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. A human nutrition researcher conducted an experiment to determine the acceptability of cakes baked with sucrose substitutes as the sweetening agent. Specifically, there were 6 recipes formed by combinations of 3 sweeteners and 2 leavening agents:

$S_1: 100\%$ sucrose $S_2: 75\%$ corn syrup, $25\%$ sucrose $S_3: 75\%$ fructose, $25\%$ sucrose

$L_1: $ Baking soda $L_2: $ Baking soda plus “additional acid”

A panel of 6 taste testers were used to evaluate various characteristics of the cakes. On each of three days, cakes were baked from all six recipes. On each day, all six tasters evaluated a sample of cake baked using each of the six recipes. The only difference between the three days was the order in which the tasters evaluated the recipes. The following table provides the tasting regimen for the three days:
1. Type of Randomization:

2. Type of Treatment Structure:

3. Identify each of the factors as being Fixed or Random:

4. Describe the experimental units and/or measurement units:

5. Identify any covariates:

6. An ANOVA Table

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>DF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

II. (10 points) A simulation model of an inventory system involved five independent variables: A, B, C, D, E, each having 2 levels. The study design was a $2^{5-2}$ fractional factorial using $I = ABDE$ and $I = BCDE$ to generate the treatments to be used in the study.

a. What is the resolution of this design? Justify your answer.

b. What is a major problem with using this design? Suggest an improved design.

III. (30 pts.) A structural engineer is studying the strength of aluminum alloy purchased from three vendors. Each vendor submits the alloy in standard-sized bars, either 1, 2, or 3 inches. The processing of different sizes of bar stock from a common ingot involves different forging techniques, and this factor may be important. Furthermore, the bar stock is forged from ingots made in different batches. Each vendor submits two test specimens of each size bar stock from 3 batches. The strength of each bar is determined and is reported in the following table. The three vendors are the only vendors under consideration, the batches are randomly selected and are representative of the batches produced by the vendors.

<table>
<thead>
<tr>
<th>Vendor</th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Batch</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>1 Inch</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>12.30</td>
<td>13.46</td>
<td>12.35</td>
</tr>
<tr>
<td>Bar Size</td>
<td>12.59</td>
<td>14.00</td>
<td>12.60</td>
</tr>
<tr>
<td>2 Inches</td>
<td>13.16</td>
<td>13.29</td>
<td>12.50</td>
</tr>
<tr>
<td></td>
<td>13.00</td>
<td>13.62</td>
<td>12.39</td>
</tr>
<tr>
<td>3 Inches</td>
<td>12.87</td>
<td>13.46</td>
<td>12.73</td>
</tr>
<tr>
<td></td>
<td>12.92</td>
<td>13.82</td>
<td>12.15</td>
</tr>
</tbody>
</table>
A. Complete the following AOV table.

<table>
<thead>
<tr>
<th>SOURCE</th>
<th>DF</th>
<th>MS</th>
<th>EMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAR SIZE</td>
<td>1</td>
<td>0.1263</td>
<td></td>
</tr>
<tr>
<td>VENDOR</td>
<td>1</td>
<td>0.4424</td>
<td></td>
</tr>
<tr>
<td>SIZE*VENDOR</td>
<td>1</td>
<td>0.0594</td>
<td></td>
</tr>
<tr>
<td>BATCH(VENDOR)</td>
<td>1</td>
<td>1.6702</td>
<td></td>
</tr>
<tr>
<td>SIZE*BATCH(VENDOR)</td>
<td>1</td>
<td>0.0919</td>
<td></td>
</tr>
<tr>
<td>ERROR</td>
<td>1</td>
<td>0.0404</td>
<td></td>
</tr>
</tbody>
</table>

B. At the $\alpha = 0.05$ level, evaluate the effect of Vendor and Bar Size on the strength of the aluminum alloy.

C. Using the numeric values of the MS’s given above and your EMS’s, find the values of the following quantities:

1. The estimate of the standard error of the estimated mean strength of Vendor 1’s alloy:
2. The estimate of the standard error of the estimated difference in the mean strength of two Vendors:
3. The estimate of the standard error of the estimated difference in the mean strength of two Bar Sizes for a given Vendor:

IV. (30 pts) Place the LETTER of the best answer in the blank to the left of each question.

(1) In a Crossover Design, both the sequence effect and the carryover effect were highly significant ($p−value < .0001$). The treatment effect

- A. cannot be tested because it is completely confounded with the carryover effect.
- B. cannot be tested because it is completely confounded with the sequence effect.
- C. cannot be tested because it is completely confounded with both the sequence and carryover effect.
- D. should be tested using only of the data observed during the first time period of each sequence.
- E. all of the above

(2) A covariate was measured along with the responses within a completely randomized design. The researcher determines that the slopes of the 5 treatment lines are different. A comparison of the 5 treatments

- A. cannot be conducted because there is an interaction between the covariate and treatment which violates the conditions for analysis of covariance.
- B. could be made using Tukey’s HSD on the sample treatment means.
- C. could be made using adjusted treatment means at specified values of the covariate.
- D. could be made using Tukey’s HSD on the adjusted treatment means.
- E. none of the above

(3) A three factor experiment is run within Factor A-fixed, Factor B-fixed nested with Factor A, Factor C-fixed crossed with Factor A. The interaction between factors A and C was not significant and the interaction between factors B(A) and C was not significant.

- A. A comparison of the differences in the levels of Factor B can be done using Tukey’s HSD on the means for the levels of Factor B averaged over the levels of Factors A and C.
B. A comparison of the differences in the levels of Factor B can be done using Tukey’s HSD on the means for the levels of Factor B computed separately at each level of factors A and C.

C. A comparison of the differences in the levels of Factor B can be done using Tukey’s HSD on the means for the levels of Factor B computed separately at each level of factor A.

D. A comparison of the differences in the levels of Factor B can be done using Tukey’s HSD on the means for the levels of Factor B computed separately at each level of factor C.

E. none of the above

(4) A RCBD experiment is conducted with five treatments. The researcher conducts both an F-test for testing for treatment differences and Friedman’s rank based test. The F-test yields a p-value of .234 whereas Friedman’s test yields a p-value of .027. The most probable reason for the difference in the conclusions reached by these two tests is

A. the researcher made a mistake in computing the value of the F-test because the F-test is always more powerful than a rank based procedure.

B. the Friedman’s test was improperly applied to a situation in which the population distributions were normally distributed.

C. the data was highly correlated so the condition of independence which is required by the F-test was violated.

D. the five population distributions were very heavy-tailed.

E. none of the above

(5) In a split-plot analysis of a repeated measures design, we consider the 5 times at which the measurements are taken to be the split-plot factor. The analysis yields only an approximation to the correct p-values because

A. the condition of compound symmetry must also be valid.

B. the experimental units are not split into 5 subunits with the levels of the time factor then randomly assigned to the subunits.

C. there is a strong correlation between the repeated measurements taken on the same experimental unit.

D. all of the above

E. none of the above

(6) Condition required in ANOVA models which if violated is most likely to result in the ratios of Mean Squares to not have an F-distribution.

A. Improper randomization and/or correlated responses from experimental units.

B. non-normal treatment population distributions.

C. treatment population distributions having unequal variances.

D. the violations listed in A, B, and C have equal impact on the F-distribution.

E. none of the above, the F-tests are very robust to the departure from the ANOVA conditions.

(7) In a Balanced Incomplete Block Design,

A. every pair of treatments appear in every block.

B. every pair of treatments appear together in every block.

C. every pair of treatments appear together in the same number of blocks.

D. every pair of treatments are paired.

E. none of the above
A researcher is studying the absorption times of two types of antibiotic capsules: T₁, T₂. There are three capsule wall thicknesses of interest to the researcher: 2, 4, .6 mm. The researcher randomly selects a wall thickness and type of capsule places this setting on the capsule formulation (FORM) machine, and produces a capsule. The researcher then determines the absorption time for the capsule. A pair of contrasts which test the null hypothesis of no interaction in terms of the cell means: μ₁₁, μ₁₂, μ₁₃, μ₂₁, μ₂₂, μ₂₃ is

A. \[ L₁ = \mu₁₁ - \mu₁₂ + \mu₂₁ - \mu₂₂ \] and \[ L₂ = \mu₁₁ + \mu₁₂ + \mu₁₃ - \mu₂₁ - \mu₂₂ - \mu₂₃ \]
B. \[ L₁ = \mu₁₁ - \mu₁₂ + \mu₂₁ - \mu₂₂ \] and \[ L₂ = \mu₁₁ + \mu₁₂ - 2\mu₁₃ + \mu₂₁ + \mu₂₂ - 2\mu₂₃ \]
C. \[ L₁ = \mu₁₁ + \mu₂₁ + \mu₁₃ - \mu₂₁ - \mu₂₂ - \mu₂₃ \] and \[ L₂ = \mu₁₁ + \mu₁₂ - 2\mu₁₃ + \mu₁₃ + \mu₂₁ + \mu₂₂ - 2\mu₂₃ \]
D. \[ L₁ = \mu₁₁ - \mu₁₂ - \mu₂₁ + \mu₂₂ \] and \[ L₂ = \mu₁₁ + \mu₁₂ - 2\mu₁₃ - \mu₂₁ - \mu₂₂ + 2\mu₂₃ \]
E. none of the above

In general, the method for estimating variance components in a mixed model which yields the most efficient estimators is

A. ANOVA moment matching estimators.
B. restricted maximum likelihood estimators.
C. adjusted least squares estimators.
D. Satterthwaite modified moment matching estimators.
E. the estimators perform equally well if all model conditions are valid.

A chemical company wishes to study the difference in response times (in milliseconds) for a number of different types of circuits used in an automatic value shutoff mechanism. From past studies, the value of \( \sigma_e \) is taken to be 2 milliseconds. The researcher has a list of over 100 Types of Circuits that are of interest to the company. The company wants to determine if there is a significant variation in the performance of the 100 Types of circuits. In order to control for the variation within each Type of Circuit, she decides it is necessary to evaluate 6 circuits of each Type selected for the study. How many different Types of Circuits must be selected for use in the experiment in order to obtain an \( \alpha = .01 \) test having power of at least 0.80 whenever the standard deviation in the Types of circuits is greater than 2.5 milliseconds.

A. 9
B. 5
C. 20
D. 15
E. cannot be determined from the given information
May 3, 2002

I. (24 points) A metallurgist for a large steel corporation designed a study to investigate the strength of three alloys \(A_1, A_2, A_3\) subjected to four different heat treatment temperatures \(1000\,^\circ C, 1200\,^\circ C, 1400\,^\circ C, 1600\,^\circ C\). There were twelve furnaces available for the study. Three furnaces were randomly assigned to each of the four temperatures. One specimen of each of the three alloys were placed in each of the twelve furnaces. Two weeks after the alloy specimens were removed from the furnaces, the tensile strength of the alloys were recorded. Thus, for each furnace there is a tensile strength measurement for each of the alloys yielding a total of 36 measurements.

(1) Give your suggested ANOVA table for this experiment, showing sources of variation and degrees of freedom. Indicate for each factor whether the factor is a fixed or random effect. Find the expected mean square for each source of variation in your ANOVA table. Indicate the appropriate denominator of the \(F\) statistic for each source.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Expected MS</th>
<th>Den. F-Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(2) Suggest any post-ANOVA analysis which would assist the metallurgist in understanding the effect of heat treatment temperature on the tensile strength of the alloys. The metallurgist’s goal is to find the combination of alloy and temperature yielding the maximum tensile strength. What type of procedure may be useful to the metallurgist in this analysis?

(3) The metallurgist decided to repeat the study but there were only four furnaces available for the second study. In order to obtain the same number of replications as in the previous study, he randomly assigned a single furnace to each temperature and then placed three specimens of each alloy in the furnace and performed the heat treatment. He thus obtained 36 measurements, three of each alloy-temperature combination. Would this study have the same ANOVA as in the study described in part (1)? If not, how would it be different? Are there any problems associated with the second study?

II. (20 points) A study was designed to compare the effect of a vitamin E supplement on the growth of guinea pigs. There are 15 guinea pigs available. The guinea pigs are randomly assigned to one of three levels of vitamin E (5 guinea pigs to each group). For each animal the body weight was recorded at the end of weeks 1, 2, 3, 4, 5, 6, and 7. All animals were given identical diets during the first two weeks. At the beginning of week 3, the vitamin E therapy was started. The three treatment levels (dose of vitamin E) were zero, low, and high dose. The data include the response variable WEIGHT for each of the 7 weekly weighings (105 observations).

(1) Give your suggested ANOVA table for this experiment, showing sources of variation and degrees of freedom. Indicate for each factor whether the factor is a fixed or random effect. Find the expected mean square for each source of variation in your ANOVA table. Indicate the appropriate denominator of the \(F\) statistic for each source.
(2) Suggest any post-ANOVA analysis which would assist the researcher in understanding the effect of vitamin E on weight gain in guinea pigs.

III. (12 points)

1. The following model was fit to the experimental data:

\[ y_{ijkm} = \mu + \tau_i + b_{j(i)} + \gamma_k + (\tau \gamma)_{ik} + (b \gamma)_{ji(k)} + e_{ijkm}, \]

where \( \mu, \ \tau_i, \ \gamma_k, \) and \((\tau \gamma)_{ik}\) are population parameters; and \(b_{j(i)}, \ (b \gamma)_{ji(k)}, \) and \(e_{ijkm}\) are independent rv's with \(N(0, \sigma^2_{B(A)}), \ N(0, \sigma^2_{B(A)C}), \) and \(N(0, \sigma^2_B)\) distributions, respectively. Complete the following AOV table for the experiment by filling in the degrees of freedom, EMS, and compute the value of the F test for each source of variation.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Expected MS</th>
<th>Den. F-Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>72.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(A)</td>
<td>44.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>22.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A*C</td>
<td>6.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B(A)*C</td>
<td>9.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>5.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Compute the variance of the following difference in two treatment means. Provide an estimate of the variance and the degrees of freedom of the estimate.

\[ \bar{y}_{1.1} - \bar{y}_{2.1}. \]

IV. (16 pts.) For each of the following questions, select ONE letter from the list at the bottom of the page which is the BEST solution to each of the following situations. Place your selection in the space to the left of each situation.
SITUATION:

...... 1. Comparing the only 2 levels of a significant qualitative factor, where this factor is not involved in any significant interactions.

...... 2. Comparing the levels of the factor TEMP having levels 40°C, 50°C, 60°C, 70°C in a CRD with no other treatment factors.

...... 3. Comparing 8 methods of determining the Fluoride content in city water samples in a BIBD in which \( \lambda = 4 \).

...... 4. In an experiment having factors A-qualitative and B-quantitative, the A*B interaction was found to be significant. The experimenter wants to test levels of A, ignoring B.

...... 5. In an experiment having factors A-qualitative and B-quantitative, the A*B interaction was found to be significant. The experimenter wants to test levels of B, ignoring A.

...... 6. The experimenter wants to test several contrast selected on the basis of an examination of the data after the experiment was completed.

...... 7. In an experiment having factors A-qualitative and B-quantitative, the A*B interaction was found to be significant. The experimenter wants to test levels of B at each level of A.

...... 8. In an experiment having factors A-qualitative and B-quantitative, the A*B interaction was found to be significant. The experimenter wants to test levels of A, at each level of B.

TECHNIQUE:

A. One of the various multiple comparison procedure
B. Trend analysis using Bonferroni contrasts
C. Trend analysis using Scheffe contrasts
D. Comparison of marginal means is not appropriate.
E. Scheffe’s technique
F. AOV moment matching
G. Dunnett’s comparison technique
I. Hsu’s comparison technique
J. REML
K. Nothing new is learned beyond the results of the F-tests from the AOV table.
L. None of the above methods are appropriate.

V. (28 points) Give short (at most 20 words) but complete answers to the following questions.

1. In a crossover design, what is the Sequence effect attempting to measure? What is the Time Period effect attempting to measure?

2. In a Balanced Incomplete Block Design, what is Balanced and what is Incomplete?

3. An experiment involved 5 factors: \( F_1, F_2, F_3, F_4, F_5 \), each at two levels but only 8 EU’s were available for the experiment. The 8 treatments were selected using the generators: \( I_1 = F_1 F_3 F_5 \) and \( I_2 = F_2 F_3 F_4 \). What is the resolution of the design and what effects are confounded with the main effect of \( F_1 \)?
4. What conditions are placed on the covariate in the use of analysis of covariance to estimate and test for treatment differences.

5. Draw a profile plot (interaction plot) for a $2^2$ factorial experiment in which the main effect of one factor is not significant but the main effect of the other factor and the interaction are significant.

6. In a $2^{6-1}$ fractional factorial design, the 6-fold interaction was used to determine which treatments would be present in the experiment. What is the resolution of this design? Justify your answer.

7. In a split-plot analysis of the repeated measures design, we consider the time at which the measurements are taken to be the split-plot factor. The analysis is considered to be only an approximation to the correct analysis. Why is the repeated measures design not a valid split-plot design?
I. (30 points) A surgeon carried out an experiment to study the effect of hospital environment on the recovery times of heart surgery patients. The outcome variable, $y$, was the number of hours that a given patient spent in an intensive care unit (ICU) after surgery. The experimental factors of interest were the type of heart surgery (at 3 levels: “single bypass,” “multiple bypass,” and “other”); and four randomly selected “environments” established in the ICU (4 levels, determined by different amounts of sound and decoration in the ICU). Within a large group of “single bypass” patients, twenty patients were selected at random, with five of the patients assigned at random to each of the four “environment” treatments. Similar random selection and assignment was carried out for the “multiple bypass” and “other” surgery groups.

(a) Give your suggested ANOVA table for this experiment, showing sources of variation and degrees of freedom. Indicate for each factor whether the factor is a fixed or random effect. Find the expected mean square for each source of variation in your ANOVA table. Indicate the appropriate denominator of the $F$ statistic for each source.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Expected MS</th>
<th>Den. F-Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Error</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>20</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) In addition to the above-described information on “Hours,” “Surgery type” and “Environment,” we also have available a fourth variable, denoted “Health Status.” This is a numerical assessment by the surgeon of the patient’s general pre-surgery health, excluding the heart condition. This “health status” variable is recorded on a scale from 0 (very bad) to 100 (very good). The “Health Status” variable is considered a possible predictor of the subsequent time spent in the ICU: a generally healthy person is likely to require less time in the ICU. Explain briefly how you could incorporate this “Health Status” information into your analysis. Write down a relevant model, and state clearly and carefully the assumptions you are making with this model.

(c) Suppose that while you were planning the experiment, the surgeon had stated, “Most of our patients are out of the ICU within 8 hours, but a few will be in there for several days.” Would knowing this information have caused you to have concerns about the analysis carried out in part (a)? If not, explain why not. If you would have concerns, state them briefly, and explain how you would modify or expand your analysis to address these concerns.

II. (30 points) In each of the following three situations, list (1) Experimental design, (2) Number of levels of each factor, (3) Number replications, (4) Whether factor is fixed or random.

A. Six female rabbits were assigned at random to 3 of the 6 different sequences of dietary treatments: $D_1, D_2, D_3$. Each rabbit receives 3 different treatments in time (during periods I, II, III). Thus there are 2 rabbits assigned to each of the three different treatment sequences. The response variable is weight gain during each period.
B. Eight different glazes are applied to clay pots. Since the kiln is variable from day to day, 8 days are used as a blocking factor. In addition, there are important differences in location inside the kiln. Eight locations are identified in the kiln. The experiment is conducted such that all 8 glazes are used once each day and such that each kiln location has 8 different glazes over the 8 days.

C. An experiment was performed to examine the effects of four chemical additives and three storage temperatures on preventing the growth of bacteria in sausages. Ten batches of sausage meat were used. Each batch of meat was divided into four equal parts, and the four parts of each batch were randomly assigned to the four additives. After mixing, each part was made into three sausages which were randomly assigned to the three storage temperatures. The measured response was the bacteria count for each sausage at the end of three months of storage.

III. (10 points) An industrial quality control study identified five factors which may have an effect on the variability in an important product characteristic. In order to determine which of these five factors has a significant impact on the product characteristic, a $2^{5-2}$ fractional factorial experiment was designed using the generating equations:

$$I = F_2 F_4 F_5$$  \hspace{1cm}  $$I = F_1 F_3 F_4$$

a. List the 8 treatments to be used in the experiment
b. What is the resolution of this design? Justify your answer.
c. List the effects confounded with the main effect of factor $F_1$: 

IV. (10 points)

1. The following model was fit to the experimental data:

$$y_{ijkl} = \mu + a_i + \tau_j + \gamma_k + (a\tau)_{ij} + (a\gamma)_{ik} + (\tau\gamma)_{jk} + (a\tau\gamma)_{ijk} + e_{ijkl},$$

with $i = 1, 2, 3$; $j = 1, 2, 3, 4$; $k = 1, 2, 3, 4, 5$; $l = 1, 2, 3, 4, 5, 6$

where $\mu$, $\tau_j$, $\gamma_k$, and $(\tau\gamma)_{jk}$ are population parameters; and $a_i$, $(a\tau)_{ij}$, $(a\gamma)_{ik}$, $(a\tau\gamma)_{ijk}$ and $e_{ijkl}$ are independent rv’s with $N(0, \sigma^2_A)$, $N(0, \sigma^2_{AB})$, $N(0, \sigma^2_{AC})$, $N(0, \sigma^2_{ABC})$, and $N(0, \sigma^2_e)$ distributions, respectively. Complete the following AOV table for the experiment by filling in the degrees of freedom, and compute the value of the F test for each source of variation.
2. Compute the variance of the following difference in two treatment means. Provide an estimate of the variance and the degrees of freedom of the estimate.

\[ \bar{y}_{11} - \bar{y}_{21}. \]

V. (20 points) **INSTRUCTIONS** Write the **ONE** letter from the second column which **BEST** matches the statement in the first column. Note, there may be multiple correct correct responses and there may be items in the second column which are unused. An item in the second column can be used only once.

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>MS</th>
<th>EMS</th>
<th>F-Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>72.2</td>
<td>1σ₂  +  rσ₂ ABC + cσ₂ AB + bσ₂ AC + bcrσ₂ A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>44.7</td>
<td>1σ₂  +  rσ₂ ABC + crσ₂ AB + acrQ B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>22.3</td>
<td>1σ₂  +  rσ₂ ABC + brσ₂ AC + abrQ C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AB</td>
<td>8.9</td>
<td>1σ₂  +  rσ₂ ABC + crσ₂ AB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AC</td>
<td>6.5</td>
<td>1σ₂  +  rσ₂ ABC + brσ₂ AC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td>1.8</td>
<td>1σ₂  +  rσ₂ ABC + arQ BC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ABC</td>
<td>3.8</td>
<td>1σ₂  +  rσ₂ ABC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Error</td>
<td>5.8</td>
<td>1σ₂</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| ......1. Method for evaluating correlation in data | A. Cross-over Design |
| ......2. Condition required in Split-Plot analysis of Repeated Measures Design | B. Least squares estimation |
| ......3. Test for heteroscedasticity of variances | C. Hartley’s F_MAX |
| ......4. Design in which each EU is observed under all t treatments | D. Log(Y+c) |
| ......5. Objective technique to find the “best” transformation | E. Shapiro-Wilks test |
| ......6. Transformation when response is a percentage | F. Runs test on Residuals |
| ......7. The additional condition needed (beyond regression) required by analysis of covariance | G. Analysis of Covariance |
| ......8. Test for normality | H. Hartley’s F-max test |
| ......9. Condition in ANOVA models which, when it is not true, has greatest effect on the distribution of the F-test | I. Compound Symmetry |
| ......10. A method of analysis of a CRD when responses are non-normal | J. Normality of residuals |
| | K. Bonferroni F-test |
| | L. Box-Cox transformations |
| | M. Equal variance of treatments |
| | N. Levine’s test |
| | O. Normality |
| | P. Split-Plot Design |
| | Q. Arsin of square root of Y |
| | R. Maximum Likelihood |
| | S. Independence of residuals |
| | T. Kruskal-Wallis test |
| | U. Equality of slopes across treatments |
| | V. Scheffe’s test |
| | W. Fisher’s protected LSD |
| | X. Latin Square Design |
| | Y. Friedman’s test |
| | Z. REML |
May 10, 2000

I. (60 points) For the following four experiments, provide the following information:

1. Type of Randomization, for example, CR, RCB, LS, Split-Plot, BIB, Crossover, etc;
2. Type of Treatment Structure, for example, single factor, crossed, nested, etc;
3. Identify each of the factors or blocking variables as being fixed or random;
4. Describe the experimental units.
5. An ANOVA Table, Including : Sources of variation, Degrees of freedom, Expected mean squares, Denominator of the F-statistic for testing each relevant effect.

A. An engineer conducted an experiment to investigate the strength of ceramic components made with three different percentages of silicon and two different cooling times. On each day of the experiment, one batch of ceramic mixture was prepared with each of the 3 percentages of silicon. The three batches were baked in a furnace and as each batch was removed, it was poured into 2 component molds, with one randomly assigned to a “slow-cooling” and the other to a “fast cooling.” After a fixed period of time after complete cooling, the strength of each component (in psi) was measured. The experimental setup was repeated on each of 10 days.

B. A seafood scientist conducted an experiment to study the drained weight of oysters after the oysters had been in cold storage. She was interested in the effects of three temperatures (0°F, 10°F, 20°F) in the cooler and length of time (1,2,3,4,5 days) the oysters were in cold storage on the variation in drained weight. Oysters were used from 6 different batches. Sufficient containers of oysters were prepared so one could be tested at each storage temperature-storage time combination from each batch. The scientist also believed that drained weight would be a function of the glycogen present in the container at the beginning of storage. Therefore, as each container of oyster was prepared, the glycogen was measured and recorded. The drained weight of the oysters in the containers was recorded as the oysters were removed from the cooler.

C. A medical specialist wants to compare two different methods (M1, M2) for treating a particular illness. She will use 8 hospitals for the study. Because there may be differences in the response between hospitals, she will block on hospitals. Each hospital has four wards of patients. She will randomly select four patients in each ward to take part in the study. Within each hospital, the wards are segregated by the sex of the patient, with two wards for female patients and two for male patients. Within each hospital, one ward of female patients and one ward of male patients will be randomly selected to receive method M1. The other wards will receive method M2. All patients in a ward receive the same treatment. Values of the response variable will be recorded for each of the 128 patients.

D. Glass lenses were produced on a production line that required 10 handling stations. The company was concerned about scratched-lens damage, which might occur at any one of the 10 stations. Two trays of lenses were randomly selected at each of the 10 stations. Each tray contained 576 lenses. A random sample of 64 lenses was selected from each tray and the lenses were inspected at two positions (inner and outer sections). The total number of defective lenses found was recorded, separately for each position, for each tray.

II. (24 points) For each of the following statements, decide if the statement is true or false and place a T or F to the left of each statement. If the statement is false explain in 10 words or less WHY you think it is false.

1. The experimenter conducts the F-test for an interaction in a CR factorial experiment involving two factors A and B and finds a significant interaction. She then selects many contrasts in the levels of the quantitative factor A at each level of the qualitative factor B. The valid method for the experimenter to test whether the contrasts are significant is to use Scheffe’s technique.
2. A RCB design with 10 blocks and a 3x4 fixed effects treatment structure is run. The homogeneity of variances condition can be evaluated using Levine’s test by considering the data as being from 12 populations, one for each treatment.

3. In a CR design with treatments, five temperatures, 10°C, 20°C, 25°C, 30°C, 35°C, and ten replications per treatment, we can decompose SST into four independent sum of squares using orthogonal polynomial contrasts.

4. After conducting the F-tests for main effects and interactions in a CR factorial experiment involving two factors A-random and B-random, the experimenter finds that the two-way interaction, A*B is significant. An appropriate follow-up technique would be to define several contrasts involving the levels of factor A, and test them separately at each of the levels of B.

5. The randomization in a Crossover Design differs from the randomization in a Randomized Complete Block (RCB) Design in that in the RCB design the EU’s are the same size for all treatments, whereas in the crossover design there are several sizes of EU’s.

6. Suppose we run a CR Design with the treatments being randomly selected levels. In computing the power of the F-statistic for testing for a treatment effect, the non-central F-distribution should be used in the computation.

7. In a factorial experiment with two factors A and B, both of which have random effects, it is not possible for both the main effects to be non-significant whenever the interaction is significant.

8. The order in which the treatments are applied to the EU’s in a crossover design is not important if we have one more period than number of treatments since any carryover effects can be estimated and the treatment means adjusted.

III. (16 points)

1. The following model was fit to the experimental data:

   \[ y_{ijkl} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij} + c_{k(j)} + (\tau c)_{ik(j)} + e_{l(ijk)}, \]

   with \( i = 1, 2; \ j = 1, 2, 3; \ k = 1, 2, 3; \ l = 1, 2 \)

   where \( \mu, \tau_i, \gamma_j, \) and \( (\tau\gamma)_{ij} \) are population parameters; and \( c_{k(j)} \), \( (\tau c)_{ik(j)} \) and \( e_{l(ijk)} \) are independent rv’s with \( N(0, \sigma^2_{C(B)}) \), \( N(0, \sigma^2_{A*C(B)}) \), and \( N(0, \sigma^2_e) \) distributions, respectively. The following AOV table was obtained in the experiment.

<table>
<thead>
<tr>
<th>Source</th>
<th>df</th>
<th>MS</th>
<th>EMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>72.2</td>
<td>( \sigma^2_e + 2\sigma^2_{A*C(B)} + 18Q_A )</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>44.7</td>
<td>( \sigma^2_e + 2\sigma^2_{A*C(B)} + 4\sigma^2_C(B) + 12Q_B )</td>
</tr>
<tr>
<td>A*B</td>
<td>2</td>
<td>22.3</td>
<td>( \sigma^2_e + 2\sigma^2_{A<em>C(B)} + 6Q_{A</em>B} )</td>
</tr>
<tr>
<td>C(B)</td>
<td>6</td>
<td>6.5</td>
<td>( \sigma^2_e + 2\sigma^2_{A*C(B)} + 4\sigma^2_C(B) )</td>
</tr>
<tr>
<td>A*C(B)</td>
<td>6</td>
<td>1.8</td>
<td>( \sigma^2_e + 2\sigma^2_{A*C(B)} )</td>
</tr>
<tr>
<td>Error</td>
<td>18</td>
<td>2.3</td>
<td>( \sigma^2_e )</td>
</tr>
</tbody>
</table>

Compute the estimates of all the variance components and proportionally allocate them to the total.

2. Compute the variances of the following differences in the treatment means. Then provide estimates of the variances and the degrees of freedom of the estimates in terms of the mean squares from the AOV table for this design.

   a. \( \bar{y}_{11..} - \bar{y}_{21..} \)

   b. \( \bar{y}_{11..} - \bar{y}_{12..} \)

17
I. (30 points) An entomologist was studying the effect of two insecticides (labeled insecticides A and B) on four species of ants (labeled species 1, 2, 3 and 4). The entomologist anticipated that approximately 50% of each species would survive twenty-four hours’ exposure to either of the insecticides.

A total of 2000 ants were obtained from each of the four species. For species 1, the ants were randomly divided into groups of 200. Each group was placed in a separate flask, resulting in a total of ten flasks filled with 200 ants each. Five of the ten flasks were randomly selected and then exposed to insecticide A. The other five flasks were exposed to insecticide B.

The same general procedure also was applied independently to each of species 2, 3 and 4.

After twenty-four hours of exposure to the selected pesticide, the entomologist examined each flask and recorded

\[ y_{ijk} = \text{Number of ants (out of 200) from species } i, \text{ insecticide } j, \text{ flask } k \]

that are still alive after 24 hours

a. Our entomologist provides you with the 40 values \( y_{ijk} \), \( i = 1, 2, 3, 4; \) \( j = 1, 2; \) \( k = 1, 2, 3, 4, 5. \) Give your suggested ANOVA table for this experiment, showing sources of variation and degrees of freedom. Indicate for each source whether the source is a fixed or random effect. Find the expected mean square for each source of variation in your ANOVA table. Indicate the appropriate denominator of the \( F \) statistic for each relevant test.

b. Write down the model for \( y_{ijk} \) associated with your ANOVA table in (a).

c. Give a brief interpretation of the interaction term in the model in part (b). Be sure to include: (i) an algebraic explanation in terms of cell means; and (ii) a graphical illustration of two cases involving “zero interaction” and “nonzero interaction,” respectively.

d. Describe briefly two important diagnostic checks you would want to carry out before reporting analysis results from (a)-(b).

e. Now our entomologist wants to carry out a similar study involving the same two insecticides. The design is identical to the design described above, except that instead of having four species of ants, the entomologist will use four species of termites. Based on previous studies, the entomologist anticipates that about 0.1% of the termites will survive 24 hours’ exposure to either insecticide, while the other 99.9% of the termites will not. Given this new information about this new experiment, would you consider it appropriate to use the methods in (a)-(b) to analyze the new termite data? Explain why or why not.

II. (40 pts.) An engineer investigated the strength of steel made with all combinations of three types of additives \( A_1, A_2 \) and \( A_3 \) and four heat treatment times: 1000, 2000, 3000, and 4000 (minutes). Six batches of steel were made with each type of additive. From each batch four molds were filled with steel and the molds were then randomly assigned to one of the four heat treatment times. On given day, three batches of steel could be made, one for each type of additive. After the completion of the heat treatment, the yield strength of the molded steel product was measured (ksi) twice for each molding.
1. Write a model to describe the experiment described above. Completely identify all terms in your model and include all conditions (constraints or distributional) placed on the terms in the model.

2. Construct an appropriate AOV table for this experiment including the expected mean squares, F-statistics, and P-values.

3. Construct all necessary interaction plots.

4. Evaluate the distributional assumptions placed on your model.

5. Estimate any variance components in the model.

6. Evaluate any trends in the average yield strength of the steel as a function of treatment time.

7. Is there a significant difference in the average yield strength of the three types of additives?

8. What are your overall conclusions concerning the effects of the type of additive and treatment times on the average yield strength of the steel?

III. (30 pts.) An industrial engineer is studying the hand-insertion of electronic components on printed circuit boards in order to improve the speed of the assembly operation. She has designed three assembly fixtures and two workplace layouts that seem promising. Operators are required to perform the assembly, and it is decided to randomly select four operators for each fixture-layout combination. However, because the workplaces are in different locations within the plant, it is difficult to use the same operators for each layout. Therefore, the four operators randomly chosen for layout 1 are different individuals from the four operators randomly chosen for layout 2. The treatment combinations in this design are run in random order and two assemblies are recorded for each operator-treatment combination. The assembly times are measured in seconds and are given in the following table.
<table>
<thead>
<tr>
<th>Fixture</th>
<th>Layout 1</th>
<th>Layout 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Operator</td>
<td>Operator</td>
</tr>
<tr>
<td>F1</td>
<td>1 2 3 4</td>
<td>5 6 7 8</td>
</tr>
<tr>
<td></td>
<td>22 23 28 25</td>
<td>26 27 28 24</td>
</tr>
<tr>
<td></td>
<td>24 24 29 23</td>
<td>28 25 25 23</td>
</tr>
<tr>
<td>F2</td>
<td>30 29 30 27</td>
<td>29 30 24 28</td>
</tr>
<tr>
<td></td>
<td>27 28 32 25</td>
<td>28 27 23 30</td>
</tr>
<tr>
<td>F3</td>
<td>25 24 27 26</td>
<td>27 26 24 28</td>
</tr>
<tr>
<td></td>
<td>21 22 25 23</td>
<td>25 24 27 27</td>
</tr>
</tbody>
</table>

1. Write a model to describe the experiment described above. Completely identify all terms in your model and include all conditions (constraints or distributional) placed on the terms in the model.

2. Construct an appropriate AOV table for this experiment including the expected mean squares, F-statistics, and P-values.

3. Construct all necessary interaction plots.

4. Evaluate the distributional assumptions placed on your model.

5. Estimate the variance components in the model.

6. Proportion the total variation in the assembly times into all relevant variance components.

7. Estimate the standard error of the difference between the two Layouts for the same Fixture.

8. Estimate the standard error of the difference between two Fixtures at the same Layout.

9. What is your overall conclusions concerning this experiment?