Stat303: Statistical Methods

Homework 08 (07/28/2009)

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1.

2. First find z^* such that $P(Z > z^* \text{ and } Z < -z^*) = \alpha$. In this case, $\alpha = 0.05$.
   Then use
   $$p \pm z^*_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$
   to compute the confidence interval.

3. First find z^* such that $P(Z > z^* \text{ and } Z < -z^*) = \alpha$. In this case, $\alpha = 0.01$.
   Then use
   $$p \pm z^*_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$
   to compute the confidence interval.

4. Use the following formula:
   $$n = \left[\left(\frac{z^*_{\alpha/2}}{m}\right)^2 p^*(1-p^*)\right]$$
   where $p^* = 0.5$, z^* is the critical value corresponding to the confidence level C.

5. (a) Use p and m given in the problem to compute n with
   $$n = \left[\left(\frac{z^*_{\alpha/2}}{m}\right)^2 p^*(1-p^*)\right]$$
   (b) Use the computed n and the new specified p to compute m
   $$m = z^*_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} \tag{0.1}$$

6. (a)
   (b) Recall that
   $$p_{\text{congested}} - p_{\text{bypass}} \overset{\text{approx}}{\sim} N\left(\pi_{\text{congested}} - \pi_{\text{bypass}}, \frac{\pi_{\text{congested}}(1-\pi_{\text{congested}})}{n_{\text{congested}}} + \frac{\pi_{\text{bypass}}(1-\pi_{\text{bypass}})}{n_{\text{bypass}}}\right) \tag{0.2}$$
   In this case
   $$\sigma_{\text{diff}} = \sqrt{\frac{p_{\text{congested}}(1-p_{\text{congested}})}{n_{\text{congested}}} + \frac{p_{\text{bypass}}(1-p_{\text{bypass}})}{n_{\text{bypass}}}}$$

(c) $H_a : \pi_{\text{bypass}} < \pi_{\text{congested}}$. That is, $H_a : \pi_{\text{congested}} - \pi_{\text{bypass}} > 0$. Use (0.2) to compute $P(p_{\text{congested}} - p_{\text{bypass}} > 0)$

7. Similar to Problem 6.
8. Similar to Problem 6.
9. Similar to Problem 6.
10. Similar to Problem 6.
11. Similar to Problem 6.
12. Use the formula (0.1)
13. Define the group "die hard" to be sample 1. Define the group "less royal" to be sample 2.
    You have to figure out n_1, π_1 and n_2, π_2. They are given in the homework problem.
    (a) You are asked to provide X_1, and X_2. They are computed as following: $X_1 = n_1 p_1$, $X_2 = n_2 p_2$.
    (b) Compute the pooled variance to conduct hypothesis testing.

$$p_{\text{pooled}} = \frac{X_1 + X_2}{n_1 + n_2}$$