Stat303: Statistical Methods

Homework 5 (07/21/2009)

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Problem 1

(a) Solved in the notes for chapter 5.

(b) Let $n = 29$ and if probability of errors caught is 0.70 then probability of errors missed$= 1-0.70 = 0.3$. Thus $\mu_{missed} = n*(1-\pi) = 29*(1-0.7) = 29*0.3$

(c) If π (or p in hw and ebook notation)=0.7 then, $\sigma = \sqrt{(n\pi(1-\pi))} = \sqrt{29*0.7*0.3}$. If we change π (or p) σ will also change.

Problem 2 In this problem we need to change a proportion (π) to Z in order to use z-table.

(a) Suppose that the probability of correctly answering a question chosen at random from a universe of possible questions is 0.85 for John. Then the probability that John scored 80% or lower is $P(X < 0.80)$ for a test with 100 problems. We need to change X to Z to use z-table. Thus,

$$P[X < 0.80] = P\left[\frac{X-\pi}{\sqrt{\pi(1-\pi)/n}} < \frac{0.8-\pi}{\sqrt{\pi(1-\pi)/n}}\right] = P\left[Z < \frac{0.8-0.85}{\sqrt{0.85(1-0.85)/100}}\right] = P[Z < -1.4] = 0.0808$$

(b) Same as part a just use $n = 250$ instead of 100.

(c) It is given that $\sigma_1 = \sqrt{0.85(1-0.85)/100}$ for John and if $\sigma_2=\frac{\sigma_1}{2}$ then what is n? Use the following $\sigma_2 = \sqrt{0.85(1-0.85)/n}$, then solve for n in dependence of σ_2. Hence $n = \{\pi(1-\pi)\}/\sigma_2^2$.

Problem 3 Use sampling distribution of $\bar{x}$. Suppose $\sigma = 5.8$ mg and the measurements are repeated 4 times, $n = 4$.

(a) $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

(b) If $\sigma = 5.8$ then what should be n so that $\sigma_{\bar{x}} = 2.9$. Using $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ we can write $n = \left(\frac{\sigma}{\sigma_{\bar{x}}}\right)^2 = \left(\frac{5.8}{2.9}\right)^2$.

(c) Suppose the question was: What is the probability that Antonio misses the true mass by more than 2.66 mg in either direction if he makes one measurement.

We want to find: $P[|X-\mu| > 2.66] = P\left[\frac{|X-\mu|}{\sigma} > 2.66/\sigma\right] = P[|Z| > 2.66/5.8] = 2P[Z > 2.66/5.8]$

(d) If he had 8 independent samples , then we want to find $P[|\bar{X}-\mu| > 2.66] = P\left[\frac{|\bar{X}-\mu|}{\sigma/\sqrt{n}} > 2.66/(5.8/\sqrt{8})\right] = P[|Z| > 2.66/(5.8/\sqrt{8})] = 2P[Z > 2.66/(5.8/\sqrt{8})]$.

Problem 4 Note that $\bar{X} \sim N\left(\mu, \left(\sqrt{\sigma^2/n}\right)^2\right)$. So $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$.

Problem 5 Using Central Limit Theorem we have, $\bar{x} \sim N\left(\mu, \left(\frac{\sigma}{\sqrt{n}}\right)^2\right)$, then find probability that the mean number of flaws exceed 1.6 (for example) ,i.e., $P[\bar{X} > 1.6]$. So change $\bar{X}$ to Z by using $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$.

Problem 6

(a)

(b) The probability that untreated is at least 25 lbs greater than treated is the same as the probability that the random variable $\mu_{\text{untreated}} - \mu_{\text{treated}}$ is greater than 25. Note that $\bar{X}_{\text{untreated}} - \bar{X}_{\text{treated}}$ is distributed $N\left(\mu_{\text{untreated}} - \mu_{\text{treated}}, \sigma^2_{\text{untreated}}/4 + \sigma^2_{\text{treated}}/4\right)$ Using this information, we find the z-score for 25.

Problem 7

First you have to check if the condition for large sample holds to have normal distribution i.e., check if $n\pi \geq 10$ and $n(1-\pi) \geq 10$. Only when these conditions hold true compute probabilities by transforming to Z.

Problem 8 Let μ for 1 egg=65.1 and σ^2=5^2 then μ for a carton having 12 eggs is μ_{carton} =12*65.1 and σ^2 for carton is $\sigma^2_{carton} = 12 * \sigma^2$. Knowing mean and variance for a carton we can find the $P[775 < Carton < 825]$ =$P[\frac{775-\mu_{carton}}{\sigma_{carton}} < Z < \frac{825-\mu_{carton}}{\sigma_{carton}}]$.

Problem 9 same as problem 2.

Problem 10 From Central limit theorem, $\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$ so that Empirical rule works for $\bar{X}$ also, lower limit= $\mu - 3 * \frac{\sigma}{\sqrt{n}}$ and upper limit= $\mu + 3 * \frac{\sigma}{\sqrt{n}}$.

Problem 11

(a) Just find the mean and s.d. for sampling distribution of $\bar{x}$ and $\bar{y}$.

(b) Use the linear combination property that $\mu_{\bar{y}-\bar{x}} = \mu_{\bar{y}} - \mu_{\bar{x}}$ and $\sigma^2_{\bar{y}-\bar{x}} = \sigma^2_{\bar{y}} + \sigma^2_{\bar{x}}$ if X and Y are independent.

Problem 12

(a) What is the approximate distribution of the proportion p_{F} of women who worked last summer? $N\left(\pi_{\text{F}}, \pi_{\text{F}}(1-\pi_{\text{F}})/n\right)$

(b) Use the linear combination property that $\pi_{\text{M-F}} = \pi_{\text{M}} - \pi_{\text{F}}$. and $\sigma^2_{\text{M-F}} = \pi_{\text{M}}(1-\pi_{\text{M}})/n + \pi_{\text{F}}(1-\pi_{\text{F}})/n$ if p_{F} and p_{M} are independent.