Stat303: Statistical Methods

Homework 4 (07/20/2009)

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Problem 1 You are asked to find the probabilities of those events.

(a) Suppose the event is $Z < 2.01$. Then the proportion of observations from a from a standard normal distribution can be found on the Z-table: $P(Z < 2.01) = .9778$

(b) Suppose the event is $Z > 2.01$. Then the proportion of observations from a from a standard normal distribution can be found on the Z-table: $P(Z > 2.01) = 1 - P(Z < 2.01) = .9778$

(c) Suppose the event is $-1.2 < Z < 1.67$. Then the proportion of observations from a from a standard normal distribution can be found on the Z-table: $P(-1.2 < Z < 1.67) = P(Z < 1.67) - P(Z < -1.2) = .9525 - .1151$.

Problem 2

You are asked to find the Z-score.

(a) Find the z such that 20% of the observations fall below z. That is, $P(Z < z) = .2$. Therefore, $z = -0.84$.

(b) Find the z such that 30% of the observations above below z. That is, $P(Z > z) = .3$. Therefore, $z = .52$.

(c) Find the number z that is the 80th percentile. That is, That is, $P(Z > z) = .8$. Therefore, $z = 0.84$

Problem 3

First, you have to calculate the z-score for that WAIS of interest. Then find p, the probability of that z-score: $P(Z < z)$.

$$z = \frac{\text{WAIS} - \mu_{\text{WAIS}}}{\sigma_{\text{WAIS}}} \tag{0.1}$$

Problem 4

First, you have to calculate the z-score for Jose

$$z = \frac{\text{Jose's SAT score} - \mu_{\text{SAT}}}{\sigma_{\text{SAT}}} \tag{0.2}$$

It is also true that,

$$z = \frac{\text{Jose's ACT score} - \mu_{\text{ACT}}}{\sigma_{\text{ACT}}} \tag{0.3}$$

From (0.2), we obtained z. We are given μ_{ACT} and σ_{ACT}. Now use (0.4), solve for Jose's ACT score

$$\text{Jose's ACT score} = z\,\sigma_{\text{ACT}} + \mu_{\text{ACT}}$$

Problem 5

Similar to Problem 4.

Problem 6

Ignore the information given about ACT. This problem is just about SAT.

"How well must Abigail do on the SAT in order to place in the top 20% of all students?

First of all we have to find the z so that there are 20% of the observations fall ABOVE z. That is, $P(Z > z) = 0.2$. This problem is equivalent to find z so that there are 80% of the observations fall BELOW z. That is, $P(Z < z) = 0.8$. Therefore $z = 0.84$.

Once you obtained z, then solve for x_{Abigail} from the following equations. Note that μ_{SAT} and σ_{SAT} are given in the problem.

Problem 7

First you have to compute the z-score first using the mean and standard deviation given in the problem.

Then find the corresponding probability using the Z-table.

Problem 8

Problem 9

(a) Find z so that $P(Z < z) = 0.25$, this z-score gives you Q_1.
Find z so that $P(Z < z) = 0.75$, this z-score gives you Q_3.

(b) Use the following equation and solve for x's, because we know z's for both cases and μ and σ are given.

$$z = \frac{x - \mu}{\sigma} \tag{0.4}$$

(d) Note that for $X \sim N(\mu, \sigma^2)$. Then $IQR_X = s_X IQR_z$, where $Z \sim N(0, 1)$, which is known as standard normal distribution or Z-distribution.

(e) Use the formulas $z_{\min} = Q_1 - 1.5IQR$ and $z_{\max} = Q_3 + 1.5IQR$ where Q_1, Q_3, and IQR are from the standard normal distribution.

Problem 10

(a),(b) Note

$$P(\text{Blood type is O}) + P(\text{Blood type is A}) + P(\text{Blood type is B}) + P(\text{Blood type is AB}) = 1$$

(c) Choose an American and a Chinese at random, independently of each other. What is the probability that both have type O blood?
Compute the product $P(\text{Blood type is O for American})P(\text{Blood type is O for Chinese})$.

(c) Choose an American and a Chinese at random, independently of each other. What is the probability that both have type O blood?
Compute:

$$\begin{aligned}
& P(\text{Blood type is O for American})P(\text{Blood type is O for Chinese}) \\
+ \; & P(\text{Blood type is A for American})P(\text{Blood type is A for Chinese}) \\
+ \; & P(\text{Blood type is B for American})P(\text{Blood type is B for Chinese}) \\
+ \; & P(\text{Blood type is AB for American})P(\text{Blood type is AB for Chinese}).
\end{aligned}$$

Problem 11

Problem 12

$$\begin{aligned} & P(\text{Education beyond high school but no bachelors}) \\ + \quad & P(\text{No HS}) + P(\text{HS \& No Further}) + P(\text{At least Bachelors}) \\ = \quad & 1 \end{aligned}$$

Problem 13

"What is the $P(\text{at least one nonword error})$?"

That is, we look for $P(X \geq 1)$.

Note that $P(X \geq 1) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

Problem 14

(b)

$$\begin{aligned} \mu_{\text{nonword+word}} &= \mu_{\text{nonword}} + \mu_{\text{nonword}} \\ \sigma_{\text{nonword+word}} &= \sqrt{\sigma^2_{\text{nonword}} + \sigma^2_{\text{nonword}}} \end{aligned}$$

(c)

$$\begin{aligned} \mu_{\text{nonword+word}} &= \mu_{\text{nonword}} + \mu_{\text{nonword}} \\ \sigma_{\text{nonword+word}} &= \sqrt{\sigma^2_{\text{word}} + \sigma^2_{\text{nonword}} + 2\rho(\sigma_{\text{word}})(\sigma_{\text{nonword}})}, \text{ where } \rho \text{ is the correlation} \end{aligned}$$

Problem 15

$$\begin{aligned} \mu_{\text{Bearings+Rod}} &= 2\mu_{\text{Bearings}} + \mu_{\text{Rod}} \\ \sigma_{\text{Bearings+Rod}} &= \sqrt{2\sigma^2_{\text{Bearings}} + \sigma^2_{\text{Rod}}} \end{aligned}$$

Problem 16

(a) "If you choose a degree recipient at random, what is the probability that the person you choose is a woman?"

Compute row sum for female divided by grand total.

(b) "What is the conditional probability that you choose a woman, given that the person chosen received a professional degree?"

Go to row for female, find the counts of the cell of female having Professional, divide that counts by the row sum for female.

Problem 17

$$\begin{aligned} \mu &= \sum_{i=1}^{n} xP(X = x) \\ &= 1P(\text{Profit} = 1) + 1.5P(\text{Profit} = 1.5) + 2P(\text{Profit} = 2) + 4P(\text{Profit} = 4) + 10P(\text{Profit} = 10) \\ \sigma^2 &= \sum_{i=1}^{n} (x - \mu)^2 P(X = x) \\ &= (1 - \mu)^2 P(\text{Profit} = 1) + (1.5 - \mu)^2 P(\text{Profit} = 1.5) + (2 - \mu)^2 P(\text{Profit} = 2) \\ &+ (4 - \mu)^2 P(\text{Profit} = 4) + (10 - \mu)^2 P(\text{Profit} = 10) \end{aligned}$$

Now take the square root of σ^2 to obtain σ.

Problem 18

(a)

$$P(\text{win } 1200) + P(\text{win } 200) + P(\text{win } 25) + P(\text{win } 11) + P(\text{win } 0) \quad = \quad 1$$

Then

$$P(\text{win } 0) \quad = \quad 1 - \{P(\text{win } 1200) + P(\text{win } 200) + P(\text{win } 25) + P(\text{win } 11)\}$$

(b)

$$\begin{aligned}
&\mu \\
&= 1200P(\text{win } 1200) + 200P(\text{win } 200) + 25P(\text{win } 25) + 11P(\text{win } 11) + 0P(\text{win } 0) \\
&= 1200P(\text{win } 1200) + 200P(\text{win } 200) + 25P(\text{win } 25) + 11P(\text{win } 11)
\end{aligned}$$

(c)

$$\begin{aligned}
\sigma^2 &= \sum_{i=1}^{n} (x-\mu)^2 P(X = x) \\
&= (1200-\mu)^2 P(\text{win } 1200) + (200-\mu)^2 P(\text{win } 200) + (25-\mu)^2 P(\text{win } 25) \\
&+ (11-\mu)^2 P(\text{win } 11) + (0-\mu)^2 P(\text{win } 0)
\end{aligned}$$