Sample Questions for Chapter 6, 7, and 8

Summer 2009

1. The data below contain the number of defects observed on each of 50 lcd screens made by a certain manufacturer. The sample mean of the number of defects is 2.36 and the population standard deviation is 1.51.

   (a) Construct a 95% confidence interval for the mean number of defects per screen for all screens produced by this manufacturer.

   $$\bar{X} \pm Z_{.05/2}\frac{\sigma}{\sqrt{n}} = 2.36 \pm 1.96\frac{1.51}{\sqrt{50}} = (1.941, 2.779)$$

   (b) If we are very certain that the true standard deviation of the number of defects per screen is below 2, what sample size would be required so that the width of the 95% confidence interval for the mean number of defects per screen is at most 0.28? Make sure you enter a whole number below.

   The width of the 95% confidence interval is at most 0.28. Then, $m = 0.28/2 = 0.14$.

   $$n = \left(\frac{1.96 \times 2}{0.14}\right)^2 = 783.97 \approx 784$$

   (c) A quality control engineer at a particular lcd screen manufacturer is studying the mean number of defects per screen. Based on historical evidence, the mean number of defects per screen was thought to be 2. There have recently been changes to the manufacturing process, and the engineer now feels that the mean number of defects per screen may be significantly larger than 2. Conduct the appropriate hypothesis test using a 0.05 level of significance.

   H_0: $\mu = 2$ vs. H_1: $\mu > 2$.

   $$z = \frac{2.36 - 2}{1.51/\sqrt{50}} = 1.686 \quad \Rightarrow \quad p - Value = P(Z > 1.686) = 0.046$$

   We reject H_0. Then, we can conclude that the mean number of defects per screen may be significantly larger than 2.

2. A sanitation department is interested in estimating the mean amount of garbage per bin for all bins in the city. In a random sample of 40 bins, the sample mean amount was 54.4 pounds and the population standard deviation was 5 pounds. Construct 95% and 99% confidence intervals for the mean amount of garbage per bin for all bins in the city.

(a) Construct 95% confidence intervals.

$$\bar{X} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 54.4 \pm 1.96\frac{5}{\sqrt{40}} = (52.850, 55.949)$$

(b) Construct 99% confidence intervals.

$$\bar{X} \pm Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 54.4 \pm 2.5758\frac{5}{\sqrt{40}} = (52.364, 56.436)$$

(c) Consider the claim that the mean amount of garbage per bin is 56.1925 pounds. Is the following statement true or false? The decision about the claim would depend on whether we use a 95% or 99% confidence interval.

56.1925 is not in 95% CI, but in 99%. The decision depends on whether we use a 95% or 99% confidence intervals.

3. A sanitation supervisor is interested in testing to see if the mean amount of garbage per bin is different from 50. In a random sample of 36 bins, the sample mean amount was 49.9 pounds and the population standard deviation was 3.8 pounds. Conduct the appropriate hypothesis test using a 0.05 level of significance.

H_0: $\mu = 50$ vs. H_1: $\mu \neq 50$.

$$z = \frac{49.9 - 50}{3.8/\sqrt{36}} = -0.158 \quad \Rightarrow \quad p - Value = 2P(Z > |-0.158|) = 0.874$$

We fail to reject H_0. Then, we can conclude that the mean amount of garbage per bin is not different from 50

4. A facilities manager at a university reads in a research report that the mean amount of time spent in the shower by an adult is 5 minutes. He decides to collect data to see if the mean amount of time that college students spend in the shower is significantly different from 5 minutes. In a sample of 9 students, he found the average time was 4.64 minutes and the standard deviation was 0.75 minutes. Using this sample information, conduct the appropriate hypothesis test at the 0.05 level of significance.

H_0: $\mu = 5$ vs. H_1: $\mu \neq 5$.

$$t = \frac{4.64 - 5}{0.75/\sqrt{9}} = -1.44 \quad \Rightarrow \quad p - Value = 2P(T > |-1.44|) = 0.188$$

with DF = 8. Since p-Value is between 0.1 and 0.2 which is greater than the level of significant level, we fail to reject H_0. Then, we can conclude that the mean amount of time that college students spend in the shower is not significantly different from 5.

5. A university with a high water bill is interested in estimating the mean amount of time that students spend in the shower each day. In a sample of 14 students, the average time was 5.18 minutes and the standard deviation was 1.22 minutes. Using this sample information, construct a 99% confidence interval for the mean amount of time that students spend in the shower each day. Construct 99% confidence intervals.

$$\bar{X} \pm t_{\alpha/2,df} \frac{s}{\sqrt{n}} = 5.18 \pm 3.012 \frac{1.22}{\sqrt{14}} = (4.198, 6.162)$$

6. Reserve Capacity (RC) is the number of minutes a fully charged battery at 80 degrees will discharge 25 amps until the battery drops below 10.5 volts. An automobile manufacturer wants to compare the mean RC for batteries from two different suppliers. The sample mean and the sample standard deviation of supplier 1 are 80.54 and 10. Also, the sample mean and the sample standard deviation of supplier 2 are 76.75 and 11.74. Construct the appropriate hypothesis test for the difference between the mean RC for supplier 1 and the mean RC for supplier 2 with 0.05 level of significance.

H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 \neq \mu_2$.

$$t = \frac{(80.54 - 76.75) - 0}{\sqrt{10^2/50 + 11.74^2/50}} = 1.738 \quad \Rightarrow \quad p - Value = 2P(T > 1.738) = 0.09$$

with DF = 49. (Use DF=40). Since p-Value is between 0.05 and 0.10 which is greater than the level of significant level, we fail to reject H_0. Then we can conclude that the mean RC for supplier 1 is not significantly differently from the mean RC for supplier 2.

7. A hospital wishes to justify the benefits of nutrition programs for pregnant women using birth weight data from newborns. The hospital hopes to show that the mean birth weight for newborns from mothers who complete the program is higher than the birth weight for newborns from mothers who do not complete the program. A group of 14 pregnant women were randomly divided into two groups; the first group received the nutrition program and the second group did not receive the program. The resulting weights (in grams) of the newborn babies from each group are shown below. The sample mean and the standard deviation of group 1 are 2588 and 104.31 and the sample mean and the standard deviation of group 2 are 2489 and 73.52. Using this sample information, conduct the appropriate hypothesis test at the 0.05 level of significance. Assume the pooled variance.

H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 > \mu_2$.

$$S_p^2 = \frac{(7-1)104.31^2 + (7-1)73.52^2}{7+7-2} = 8142.883$$
$$t = \frac{2588 - 2489}{90.238\sqrt{1/7 + 1/7}} = 2.052$$

with DF=12. Since p-Value $= P(T > 2.052) = 0.031$ between 0.01 and 0.05, we reject H_0. Therefore, we can conclude that the mean birth weight for newborns from mothers who complete the program is higher than the birth weight for newborns from mothers who do not complete the program.

8. A professor is interested in comparing the average amount spent on textbooks for freshmen and sophomores. A random sample of 14 freshmen yielded a sample mean amount of $1067 and a sample standard deviation of $51. A random sample of 10 sophomores yielded a sample mean amount of $1279 and a sample standard deviation of $303. Construct a 99% confidence interval for the difference between the mean amount spent on textbooks by freshmen and the mean amount spent by sophomores (ie. do freshmen minus sophomores). Since the sample standard deviations are wildly different, use a method which does not assume the populations have the same variance.

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2,df}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = (1067 - 1279) \pm t_{0.005,9}\sqrt{\frac{1067^2}{14} + \frac{1279^2}{10}} = (-1820.274, 1396.274)$$

9. A consumer advocacy group feels that Walmart provides a less safe shopping environment than Target. To try to prove their point, they randomly selected 21 Walmart stores and found the annual number of crime reports filed for these stores. Each of these Walmart stores was then paired with a Target store within a 10 mile radius, and the annual crime reports for the associated Target stores were also recorded. The sample mean of Walmart is 831.05 and those of Target are 808.05. The standard deviation of difference is 45.06. Test to to see if the mean number of crime incidents at Walmart stores is significantly larger than the mean number of crime incidents at Target stores. The level of significance is 0.05

   H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 > \mu_2$.

$$t = \frac{(831.05 - 808.05) - 0}{\frac{45.06}{\sqrt{21}}} = 2.339$$

   Since p-Value $= P(T > 2.339) = 0.0149$ which is between 0.01 and 0.02, we reject H_0. We can conclude that the mean number of crime incidents at Walmart is significantly larger than those at Target.

10. A web based software company is interested in estimating the proportion of individuals who use the

Firefox browser. In a sample of 200 of individuals, 34 users stated that they used Firefox.

(a) Using this data, construct a 99% confidence interval for the proportion of all individuals that use Firefox.

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \frac{34}{200} \pm 1.96\sqrt{\frac{0.17(1-0.17)}{200}} = (0.118, 0.222)$$

(b) What sample size would be required so that the width of the 99% confidence interval would be at most 0.05 units wide?

At most 0.05 units wide $\Rightarrow m = 0.025$

$$n = \left[\left(\frac{1.96}{0.025}\right)^2 0.17(1-0.17)\right] = 867.2477 \approx 868$$

11. A research report claims that 20% of all individuals use Firefox to browse the web. A software company is trying to determine if the proportion of their users who use Firefox is significantly different from 0.2. In a sample of 200 of their users, 27 users stated that they used Firefox. Using this data, conduct the appropriate hypothesis test using a 0.1 level of significance.

H_0: $p = 0.2$ vs. H_1: $p \neq 0.2$.

$$z = \frac{27/200 - 0.2}{\sqrt{\frac{0.2(1-0.2)}{200}}} = -2.29$$

Since p-Value $= 2(P > |-2.29|) = 0.0215$, we reject H_0. Then we can conclude that the proportion of Firefox user is significantly different from 0.2

12. Company A makes a large shipment to Company B. Company B can reject the shipment if they can conclude that the proportion of defective items in the shipment is larger than 0.1. In a sample of 400 items from the shipment, Company B

nds that 44 are defective. Conduct the appropriate hypothesis test for Company B using a 0.05 level of significance.

H_0: $p = 0.1$ vs. H_1: $p > 0.1$.

$$z = \frac{44/400 - 0.1}{\sqrt{\frac{0.1(1-0.1)}{400}}} = 0.667$$

Since p-Value $= P(Z > 0.667) = 0.252$, we fail to reject H_0. Then, Company B does not reject the shipment.

13. Shortly after September 11th 2001, a researcher wanted to determine if the proportion of females that favored war with Iraq was significantly different from the proportion of males that favored war with Iraq. In a sample of 60 females, 26 favored war with Iraq. In a sample of 53 males, 34 favored war with Iraq.

(a) Construct a hypothesis test using 0.01 level of significance.

H_0: $p_1 = p_2$ vs. H_1: $p_1 \neq p_2$.

$$\begin{aligned} \hat{p} &= \frac{26+34}{60+53} = 0.531 \\ z &= \frac{26/60 - 34/53}{\sqrt{0.531(1-0.531)\left(\frac{1}{60}+\frac{1}{53}\right)}} = -2.21 \end{aligned}$$

Since p-Value $= 2P(Z > |-2.21|) = 0.0269$, we fail to reject H_0. Then, we can conclude the proportion of females that favored war with Iraq was significantly different from the proportion of males that favored war with Iraq.

(b) Construct a 99% confidence interval for the difference between the proportion of females that favor the war and the proportion of males that favor the war?

$$\hat{p}_1 - \hat{p}_2 \pm Z_{.005}\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} = (-0.445, 0.0283)$$