Sample Questions for Chapter 4 and 5

Summer 2009

1. Using the frequency table below,

Defects	0	1	2	3	4	5	6
Relative Frequency	0.124	0.32	0.204	0.096	0.116	0.092	0.048

(a) What proportion of the screens have exactly 3 defects?

$$P(X = 3) = 0.096$$

(b) What propotion of the screens have at least 3 defects?

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) = 0.096 + 0.116 + 0.092 + 0.048$$

(c) What proportion of the screens have fewer than 3 defects?

$$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2) = 0.124 + 0.32 + 0.204$$

(d) What proportion of the screens have more than 3 defects?

$$P(X > 3) = P(X = 4) + P(X = 5) + P(X = 6) = 0.116 + 0.092 + 0.048$$

(e) What proportion of the screens have at most 3 defects?

$$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 0.124 + 0.32 + 0.204 + 0.096$$

2. A college student is taking two courses. The probability she passes the first course is 0.8. The probability she passes the second course is 0.63. The probability she passes at least one of the courses is 0.91.

(a) What is the probability she passes both courses?

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ P(A \cap B) &= P(A) + P(B) - P(A \cup B) = 0.8 + 0.63 - 0.91 \end{aligned}$$

(b) Is the event she passes one course independent of the event that she passes the other course?

$$P(A \cap B) \neq P(A)P(B)$$

They are not independent.

3. Two professors are applying for grants. Professor Jane has a probability of 0.65 of being funded. Professor Joe has probability 0.21 of being funded. Since the grants are submitted to two different federal agencies, assume the outcomes for each grant are independent.

   (a) What is the probability that both professors get their grants funded?

$$P(A \cap B) = P(A) \times P(B) = 0.65 \times 0.21$$

   because A and B are independent.

   (b) What is the probability that at least one of the professors will be funded?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.65 + 0.21 - 0.65 \times 0.21$$

4. Consider the probability model given in the table below: Fill up ???

Outcome	0	1	2	3
Probability	0.005	0.114	0.101	???

$$P(X = 3) = 1 - P(X = 0) + P(X = 1) + P(X = 2) = 1 - 0.005 - 0.114 - 0.101$$

5. A company has 4 service trucks. Let X represent the number of trucks that are not working at any point in time. Consider the probability model shown below for X.

Outcome	0	1	2	3	4
Probability	0.2	0.2	0.16	0.05	0.39

   (a) What is the probability that exactly 2 of the trucks are not working at any point in time?

$$P(X = 2) = 0.16$$

   (b) What is the probability that fewer than 2 of the trucks are not working at any point in time?

$$P(X < 2) = P(X = 0) + P(X = 1) = 0.2 + 0.2 = 0.4$$

   (c) What is the probability that more than 2 of the trucks are not working at any point in time?

$$P(X > 2) = P(X = 3) + P(X = 4) = 0.05 + 0.39 = 0.44$$

(d) What is the expected number of trucks that are not working at any point in time?

$$\begin{aligned}\mu_X &= \sum_i X_i P(X = x_i)\\ &= 0 \times P(X=0) + 1 \times P(X=1) + 2 \times P(X=2) + 3 \times P(X=3) + 4 \times P(X=4)\\ &= 0 \times 0.2 + 1 \times 0.2 + 2 \times 0.16 + 3 \times 0.05 + 4 \times 0.39\end{aligned}$$

(e) What is the variance of X?

$$\begin{aligned}\sigma_X^2 &= \sum_i (X_i - \mu_x)^2 P(X = x_i)\\ &= (0-\mu_x)^2 \times P(X=0) + (1-\mu_x)^2 \times P(X=1) + (2-\mu_x)^2 \times P(X=2)\\ &+ (3-\mu_x)^2 \times P(X=3) + (4-\mu_x)^2 \times P(X=4)\\ &= (0-\mu_x)^2 \times 0.2 + (1-\mu_x)^2 \times 0.2 + (2-\mu_x)^2 \times 0.16 + (3-\mu_x)^2 \times 0.05\\ &+ (4-\mu_x)^2 \times 0.39\end{aligned}$$

(f) What is the standard deviation of X?

The Square Root of Variance

6. A company that produces DVD drives has a 13% defective rate. Let X represent the number of defectives in a random sample of 55 of their drives.

(a) What is the expected number of defective drives in the sample?

$$\mu_x = n\pi = 55 \times .13$$

(b) What is the variance of the number of defective drives in the sample?

$$\sigma_x^2 = n\pi(1-\pi) = 55 \times .13 \times (1 - .13)$$

(c) What is the standard deviation of the number of defective drives in the sample?

The squared root of variance is the standard deviation

(d) Each defective drive costs the company 15 dollars. What is the expected cost to the company for the defective drives in the sample?

$$\mu_{15x} = 15\mu_x = 15n\pi = 15 \times 55 \times .13$$

(e) Each defective drive costs the company 15 dollars. What is the variance of the cost to the company for the defective drives in the sample?

$$\sigma^2_{15x} = 15^2\sigma^2_x = 15^2 n\pi(1-\pi) = 15^2 \times 55 \times .13 \times (1-.13)$$

7. A manufacturer of industrial solvent guarantees its customers that each drum of solvent they ship out contains at least 100 lbs of solvent. Suppose the amount of solvent in each drum is normally distributed with a mean of 103.1 pounds and a standard deviation of 3.7 pounds. What is the probability that a drum meets the guarantee?

$$\begin{aligned} P(X < 100) &= P\left(\frac{X-\mu}{\sigma} < \frac{100-\mu}{\sigma}\right) = P\left(\frac{X-\mu}{\sigma} < \frac{100-103.1}{3.7}\right) \\ &= P(Z < -0.84) = 0.2004 \end{aligned}$$

8. The probability to have a A blood type is 0.5 for an American and 0.25 for a Chinese. What is the probability that if we choose an American and Chinese at random, independently of each other; that both will have A blood type? Also at least one of the will have A blood type? (Please write down whole procedure for your calculation for partial or full credit.)

A: A Blood Type in American, B: A Blood Type in Chinese

$$P(A \cap B) = P(A) \times P(B) = 0.5 \times 0.25$$

because A and B are independent.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.25 - 0.5 \times 0.25$$

9. Spell-checking software catches "nonword errors" that result in a string of letters that is not a word, as when "the" is typed as "teh". When undergraduates are asked to write a 250-word essay (without spell-checking), the number X of nonword errors has the following distribution. (Please write down whole procedure for your calculation for partial or full credit.)

X	0	1	2	3	4
p(X)	0.15	0.20	0.30	0.25	0.10

(a) Find the Mean for random variable X.

$$\begin{aligned}\mu_X &= \sum_i X_i P(X = x_i)\\ &= 0 \times P(X=0) + 1 \times P(X=1) + 2 \times P(X=2) + 3 \times P(X=3) + 4 \times P(X=4)\\ &= 0 \times 0.15 + 1 \times 0.20 + 2 \times 0.30 + 3 \times 0.25 + 4 \times 0.10\end{aligned}$$

(b) Find the Variance and Standard Deviation for random variable X.

$$\begin{aligned}\sigma_X^2 &= \sum_i (X_i - \mu_x)^2 P(X = x_i)\\ &= (0-\mu_x)^2 \times P(X=0) + (1-\mu_x)^2 \times P(X=1) + (2-\mu_x)^2 \times P(X=2)\\ &+ (3-\mu_x)^2 \times P(X=3) + (4-\mu_x)^2 \times P(X=4)\\ &= (0-\mu_x)^2 \times 0.15 + (1-\mu_x)^2 \times 0.20 + (2-\mu_x)^2 \times 0.30 + (3-\mu_x)^2 \times 0.25\\ &+ (4-\mu_x)^2 \times 0.10\end{aligned}$$

Then, the standard deviation is the square root of variance.

10. Typographic errors in a text are either nonword errors (as when "the" is typed as "teh") or word errors that result in a real but incorrect word. Spellchecking software will catch nonword errors but not word errors. Suppose human proofreaders catch 75% of nonword errors. You ask a fellow student to read an essay in which you have deliberately made 50 nonwords errors. What is the mean and variance of error caught? (Please write down whole procedure for your calculation for partial or full credit.)

$$\begin{aligned}\mu_X &= n\pi = 50 \times .75 = 37.5\\ \sigma_X^2 &= n\pi(1-\pi) = 50 \times .75 \times (1 - .75)\end{aligned}$$

Then, the standard deviation is the square root of variance.

11. Suppose past studies indicate it takes an average of 7 minutes to memorize a short passage of 20 words with population standard deviation 3. A psychologist claims a new method of memorization will reduce the average time to 4 minutes with population standard deivation 2. To check the validity of his claim, a random sample of 30 people are to used. What is the sampling distribution for current method and new method? (Assume that the average is the population mean).

X: Current Method, Y: Proposed Method

$$X \sim N\left(7, \frac{3^2}{30}\right), \qquad X \sim N\left(4, \frac{2^2}{30}\right)$$