

STAT303: Secs 102 and 103
Summer I 2000
Exam #3
Form A

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1. **Don't EVEN open this until you are told to do so.**
2. Be sure to mark your section number and your test form (A, B, C or D) on the scantron!
3. Sign your name where indicated on your scantron and write your section number, seat number and computer number beside it. You will get your scantrons back tomorrow in class. You may keep this exam.
4. There are 20 multiple-choice questions on this exam, each worth 5 points. There is partial credit. Please mark your answers **clearly** on the scantron. Multiple marks will be counted wrong.
5. You will have 60 minutes to finish this exam.
6. If you are caught cheating or helping someone to cheat on this exam, you both will receive a grade of **zero** on the exam. You must work alone.
7. This exam is worth 100 points, and will constitute 20% of your final grade.
8. Good luck!

1. Oprah wants to increase revenue. She has a lot of female targeted advertising, so her only outlet is to increase the advertising targeted at males. You need at least a 20% viewing audience to attract advertisers, so she needs to find out if her audience is at least 20% male. What hypotheses should she test? Let F indicate females and M males.

- A. $H_0 : \mu_F = \mu_M$ vs. $H_A : \mu_F \neq \mu_M$
 B. $H_0 : \pi_F = \pi_M$ vs. $H_A : \pi_F > \pi_M$
 C. $H_0 : \mu_F = \mu_M$ vs. $H_A : \mu_F > \mu_M$
 D. $H_0 : \pi_M = 0.20$ vs. $H_A : \pi_M < 0.20$
 *E. $H_0 : \pi_M = 0.20$ vs. $H_A : \pi_M > 0.20$

First, this problem is about the proportion of males. So the hypotheses should involve π_M . Furthermore, Oprah needs to find out if her audience is at least 20% male so the research(alternative) hypothesis(H_A) should be $H_A : \pi_M > 0.20$

2. Same scenario: Advertisers are tight with their money, so they want strong proof that there is at least a 20% viewing audience. What α -level would the **advertising company** want Oprah to use?

- A. $\alpha = 0.05 = 5\%$ because neither Type I nor Type II error is critical.
 B. $\alpha = 0.10 = 10\%$ because Type I is more critical.
 *C. $\alpha = 0.01 = 1\%$ because Type I is more critical.
 D. $\alpha = 0.10 = 10\%$ because Type II is more critical.
 E. $\alpha = 0.01 = 1\%$ because Type II is more critical.

Strong proof means a very small p-value. A Type I error is when we reject H_0 when H_0 is true. For this case, we would say that more than 20% of the viewing audience is male when it is indeed $\leq 20\%$. The advertisers would then spend money for no gain. A Type II error would be that the advertisers didn't spend the money (since it was a fail to reject) even though they would have gotten a large male audience (H_0 false).

3. Which of the following is true?

- A. A p-value is how often we would get data as contradictory as we got even though H_0 is true.
 B. A p-value is a measure of the strength of the evidence against the null hypothesis.
 C. A p-value can be used to perform a test of hypotheses at any significance level.
 *D. All of the above are true.
 E. Exactly two of the above are true.

A p-value is the probability, assuming that H_0 is true, of obtaining a value of the test statistic at least as extreme(contradictory) to H_0 as the value calculated from the data. It is evidence against H_0 in favor of the H_A .

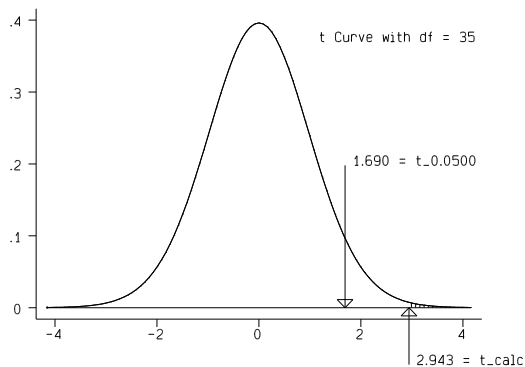
4. Which of the following is NOT a property of the t distribution?

- A. It is always centered at zero.
 The t curve, like the z curve, has a mean=0.
 B. It is always wider than the z distribution. Only if we had an infinite sample (infinite degrees of freedom) would the t look exactly like the z .
 C. It is always symmetric. Again, like the normal, the t is symmetric (actually it's also bell-shaped only with longer tails).
 D. It's height, at center, increases as the degrees of freedom increase.

The values of t curve vary with the different degrees of freedom. It is always less than that of standard z curve at 0. Furthermore, remember that t curve gets closer to z curve as the degrees of freedom increase.

- *E. All of the above are properties of the t . See the table of t distribution, t curve, and z curve.

Right-Sided Test for Mean of nonnormal, sigma unknown but n large:
 $n = 36$ $\bar{x} = 12.6$ $s^2 = 5.3^2$ $\alpha = .05$ $\mu_0 = 10$
 p-value = shaded area = 0.003



5. What is the correct conclusion that can be made from the output above which is testing $H_0 : \mu = 10$ vs. $H_A : \mu > 10$?

- *A. Reject the null at the 10, 5 and 1% levels and conclude that the true mean is greater than 10.
- B. Reject the null at the 10, 5 and 1% levels and conclude that the true mean is 10.
- C. Reject the null at the 10, 5 and 1% levels and conclude that the true mean is NOT 10.
- D. Reject the null at the 10% level ONLY and conclude that the true mean is greater than 10.
- E. Reject the null at the 10% level ONLY and conclude that the true mean is NOT 10.

Because the p-value is 0.003 which is less than 10%, 5%, and 1%, we reject H_0 in favor of H_A at each level of significance. Hence, we reject H_0 at the 10, 5 and 1% levels and conclude that the true mean is greater than 10.

6. In the test above, the sample mean, $\bar{x} = 12.6$. What would change if we had gotten $\bar{x} = 11$ instead?
- A. The conclusion would be the same, since the hypotheses are the same.
 The conclusion depends on the value of \bar{x} , s and n we get from the data. The fact that the hypotheses are the same does not affect whether or not the conclusion will change.
 - *B. The p-value would be larger since the new sample mean is closer to the hypothesized mean, $\mu = 10$.

Because the new sample mean is closer to the hypothesized mean, the z value will be closer to 0. Therefore, the p-value would be larger.

- C. The p-value would be smaller since the new sample mean is closer to the hypothesized mean, $\mu = 10$.
- D. The sign of the alternative, H_A , would change from $>$ to $<$.

We are working with the same hypotheses.

- E. It's impossible to tell without rerunning the test.

7. What do we mean by the term *confidence* in reference to confidence intervals?

- A. We are confident that our data is random.
- *B. We are confident that our method produces intervals that contain the parameter $(1 - \alpha)100\%$ of the time.
- C. We are confident that our method produces intervals that contain the parameter $\alpha * 100\%$ of the time.
 We usually consider the α to be the error rate.
- D. We are confident that our interval contains the parameter.
 The confidence level (ex.95%) refers to the *method* used to construct the interval rather than to any particular interval.
- E. We are confident that our method produces intervals that contain the statistic $(1 - \alpha)100\%$ of the time.

For example, the 95% confidence interval for μ means that, if we take sample after sample from the population and use each one separately to compute a 95% confidence interval using the same method, in the long run roughly 95% of these intervals will capture μ , the population parameter.

90% | Lower Limit = 6.1404199
| Upper Limit = 6.8595801

95% | Lower Limit = 6.0689941
| Upper Limit = 6.9310059

99% | Lower Limit = 5.9252143
| Upper Limit = 7.0747857

8. Using the information above, what is the correct range of the p -value if I wanted to test $H_0 : \mu = 5$ vs. $H_A : \mu \neq 5$?
- A. $p\text{-value} > 0.10$
 - B. $0.10 > p\text{-value} > 0.05$
 - C. $0.05 > p\text{-value} > 0.01$
 - *D. $p\text{-value} < 0.01$
 - E. You need a test statistic value to determine the p -value

Since the hypothesized value of 5 is not contained in any of these intervals, the p -value is less than all of (100-90)%, (100-95)% and (100-99)% . So, we have $p\text{-value} < 0.01$. See the problem 15 following.

9. Which of the following would define a Type I error for the test above, $H_0 : \mu = 5$ vs. $H_A : \mu \neq 5$? Type I error: reject H_0 in favor of H_A when H_0 is true. This means we would conclude the true mean is NOT 5 (reject H_0) even though it was 5 (H_0 true).
- A. We claim the true mean is 5 when it actually is NOT 5.
 - *B. We claim the true mean is NOT 5 when it actually IS 5.
 - C. We claim the true mean is NOT 5 when it actually is greater than 5.
 - D. We conclude there is insufficient evidence to claim the true mean is NOT 5 even though it actually is NOT 5.
 - E. We conclude there is insufficient evidence to claim the true mean is NOT 5 even though it actually IS 5.
10. Which of the following is a TRUE statement about hypothesis testing in general?
- *A. If we reject H_0 , we conclude that the alternative hypothesis is true.
 - B. If we fail to reject H_0 , we conclude that the null hypothesis is true.
When we fail to reject H_0 , we conclude that there is not strong

evidence that the alternative hypothesis is true. The sample does not lead to a rejection of H_0 because we calculate the value of statistic under H_0 and infer from that. We never can PROVE the null true. We assume it is true and try to find evidence against it in favor of the alternative.

- C. If we fail to reject H_0 , we conclude that the alternative hypothesis is false.
 - D. We initially assume the alternative is true and then gather data to support it.
 - E. Two of the above are true.
11. Texas A&M wants to find out whether *most* students are opposed to the proposed Rec Center fee increase from \$50 to \$60. Which of the following is the correct statement of the hypotheses that A&M should test?
- A. $H_0 : \mu = \$50$ vs. $H_A : \mu = \$60$
 - *B. $H_0 : \pi = 0.5$ vs. $H_A : \pi > 0.5$
 - C. $H_0 : \mu = 0.5$ vs. $H_A : \mu > 0.5$
 - D. $H_0 : \mu = \$10$ vs. $H_A : \mu > \$10$
 - E. $H_0 : \mu_1 - \mu_2 = \$10$ vs. $H_A : \mu_1 - \mu_2 > \$10$

This problem is about the a proportion since we are counting how many students out of all that are asked are opposed. The fact that the proportion of the opposed students is greater than 0.5 means *most* students are opposed to the proposed Rec Center fee increase.

12. Suppose A&M concluded that the students thought the increase was acceptable (less than half were opposed) when in reality they only asked students who don't use the Rec Center. Students using the Rec Center are strongly opposed to the increase. What happened?
- A. A&M took a biased sample which caused them to make a Type I error.
 - *B. A&M took a biased sample which caused them to make a Type II error.
 - C. A&M made a correct decision.
 - D. A&M tested the wrong hypotheses.
 - E. A&M will always go for the increase.

Since A&M only asked students who don't use the Rec Center, the sample is biased. Also, since the students using the Rec Center are strongly opposed to the increase, H_0 is false. A&M failed to reject since they conclude there

WASN'T a majority opposed. This means they made a Type II error.

13. Suppose you were testing $H_0 : \mu = 5$ vs $H_A : \mu > 5$ and ended up with a p-value = 0.03. Which of the following is the best interpretation of the p-value?

A. 3% of the time we would reject even though H_0 is true.

α is the proportion of the time that we reject when H_0 is true, not the p-value.

B. In repeated sampling, we would get p-values of 3% or more even though H_0 is true.

In repeated sampling with H_0 true, we would get p-values ranging from 0 to 1.

C. 3% of the time in repeated sampling, we would get sample means = 5 or more even though H_0 is true.

5 is the hypothesized value. If H_0 is true, then we would get sample means of 5 or more HALF of the time.

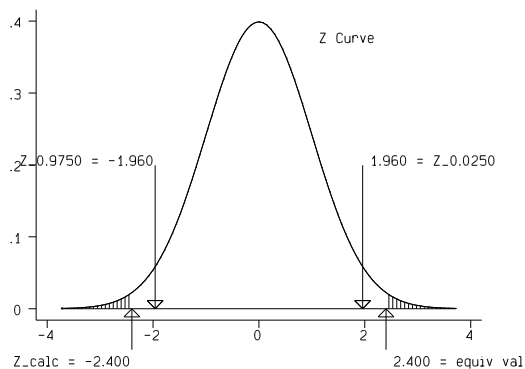
*D. 3% of the time in repeated sampling, we would get sample means at least as large as the sample mean here even though H_0 is true.

The p-value is how often (3% of the time) we get what we got or worse (at least as large since we have a $>$ alternative) even though H_0 is true (the true mean is 5).

E. 3% of the time in repeated sampling, we would make a Type I error.

This is the interpretation if $\alpha = 0.03$

Two-Sided Test for 0-1 proportion pi (approximate):
 $n = 36$ $p = .3$ $\alpha = .05$ $\text{hyp val} = .5$
 p-value = shaded area = 0.016



14. The hypotheses being tested here are $H_0 : \pi = 0.5$ vs $H_A : \pi \neq 0.5$. What is the correct conclusion?

A. $\alpha = 0.05 < 0.5$, so we reject and conclude the coin is unfair.

B. $\alpha = 0.05 < 0.5$, so we reject and conclude the coin is fair.

C. $\alpha = 0.05 < 0.5$, so we fail to reject and conclude the coin is unfair.

D. $\alpha = 0.05 < 0.5$, so we fail to reject and conclude the coin is fair.

*E. Reject since the p-value is less than α and conclude the coin is unfair.

When we decide whether we reject H_0 or not, we compare the p-value to α . 0.5 is the hypothesized value. It is used to calculate the test statistic and thus the p-value, but it is NOT ever compared to α .

15. Which of the following best describes the relationship between a $(1 - \alpha)100\%$ confidence interval for μ and a 2-sided test of hypotheses for $\mu = \text{some value}, \mu_0$?

A. There is no relationship between confidence intervals and hypothesis tests.

B. If the hypothesized value, μ_0 , falls within the confidence interval, we would reject the null.

*C. If the hypothesized value, μ_0 , falls within the confidence interval, we would fail to reject the null.

D. If the confidence interval contains 0, we would reject the null.

E. If the confidence interval contains 0, we would fail to reject the null.

This is true if $\mu_0=0$ or we're doing a 2 sample test on the difference.

Confidence intervals are ranges of plausible values, so if the hypothesized value is one of those, then we could NOT reject it and the p-value would be GREATER than α .

16. Suppose we have a biased coin so the true probability of getting a head is actually 90%, $\pi = 0.90$. How many times must we toss it for the sampling proportion, p , to be approximately normally distributed?

- A. A sample size, $n \geq 30$ is always necessary.
This is the necessary assumption for numerical non-normal data.
- B. Categorical data, like this, cannot ever be approximated by a normal.
The normal approximation is often used for binomial data.
- C. The rule for categorical data says at least 10 tosses are necessary.
The rule says $n\pi$ AND $n(1 - \pi) \geq 10$.
- D. The rule for categorical data requires we use at least 100 tosses.
 $\pi = 0.90$, but $(1 - \pi)$ is only 0.1!
- E. The rule for categorical data says 12 would be sufficient. $10/0.1 = 100$

17. Suppose $p_{50} \sim N(0.3, 0.065^2)$. How likely are we to get a sample proportion of at least 50%, i.e., what is $P(p_{50} > 0.5)$?

- A. 3.08% of the time
This is the z-score for 0.5.
- B. 99.9% of the time
This is $P(p_{50} < 0.5)$.
- *C. 0.1% of the time
 $P(p_{50} > 0.5) = P(Z > (0.5 - 0.3)/0.065) = P(Z > 3.08) = P(Z < -3.08) = 0.001$.
- D. 1% of the time
The decimal is wrong.
- E. 0.001% of the time
The decimal is wrong.

$0.5 \approx 0.3 + 3 \times 0.065$ and p_{50} is normally distributed. So almost all of the area associated with values are within three standard deviations of the mean. More accurately,

$$P(0.3 - 3 \times 0.065 < p_{50} < 0.3 + 3 \times 0.065) = 99.74\%$$

So the about half of 0.26%(1-99.74%) will be $P(p_{50} > 0.5)$.

18. Which of the following do NOT affect the width of a confidence interval for the mean, μ , of a normal population when the standard deviation, σ , is known?

- A. the sample mean
B. the sample size

- C. the sample standard deviation
D. All of the above will affect the width.
*E. Exactly two of the above (excluding D) will affect the width.

The width of the confidence interval for μ is twice the (critical value) $\times \frac{\sigma}{\sqrt{n}}$. The SAMPLE standard deviation is not needed since we know the TRUE standard deviation.

19. Using $\alpha = 0.05$ means that

- A. we will reject H_0 5% of the time.
We will reject $\alpha\%$ of the time ONLY if H_0 is true. If H_0 is false, we hope to reject alot more often.
- *B. we will commit a Type I error 5% of the time.
- C. we will be wrong (i.e., make the wrong decision) 5% of the time.
There are two kinds of wrong decision; Type I and Type II error. So the probability of making the wrong decision is $0.05(\alpha) + P(\text{Type II error})$, which is greater than 0.05.
- D. All of the above are true.
- E. Exactly two of the above are true (excluding D).

20. Suppose we sampled the population, $X \sim N(10, 3^2)$, 100 times and created 95% confidence intervals from each sample. Which of the following would be true?

- *A. About 95% of the intervals would contain 10.
B. At least 95% of the intervals would contain 10.
C. Since the true mean is 10, all of the intervals would include 10.
D. 5% of the intervals would be Type I errors.
E. 5% of the intervals would be wrong.

See the explanation for problem 7.

1E,2C,3D,4E,5A,6B,7B,8D,9B,10A,11B
12B,13D,14E,15C,16D,17C,18E,19B,20A