

Name: \_\_\_\_\_

1. (10 points) Derive a formula for  $\beta$  and  $\sigma^2$  for an MA(1) process in terms of  $R(0)$  and  $R(1)$ .
2. (10 points) If  $\epsilon \sim WN(\sigma^2)$ ,  $\alpha$  is a number in  $(-1, 1)$ ,  $X(0)$  is a  $N(0, \sigma^2/(1 - \alpha^2))$  random variable independent of  $\epsilon$ , and for  $t \geq 1$  we have

$$X(t) = \alpha X(t - 1) + \epsilon(t),$$

what is  $\text{Var}(X(1))$ ? How about  $\text{Var}(X(2))$ ? How about for general  $X(t)$ ?

3. (10 points) If the truncation point  $M$  is 6 for the Parzen lag window generator

$$\lambda(u) = \begin{cases} 1 - 6u^2 + 6|u|^3, & |u| \leq 0.5 \\ 2(1 - |u|)^3, & 0.5 \leq u \leq 1, \\ 0 & |u| > 1 \end{cases}$$

draw a graph of  $k_n(v)$  for  $|v| \leq 6$ .

4. (10 points) What is the frequency transfer function for the moving average smoother having  $2M + 1$  terms in the average?
5. (10 points) If  $X(t) = A \cos 2\pi t\omega + B \sin 2\pi t\omega$  where  $A$  and  $B$  are independent  $N(0, \sigma^2)$  random variables, show that  $X$  is covariance stationary. (Note that  $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ .)
6. (10 points) If  $(1, 2, 3)$  is a realization of length 3 from an MA(1,1,1) process, what is the BLUP of  $X(4)$ ? (Hint: you can solve the 3 by 3 system of equations via substitution. The answer should be a number.)
7. (10 points) Let  $Y(t) = \mu + X(t)$  where  $X$  is an AR(2) process with  $\alpha_1 = 0.3$ ,  $\alpha_2 = 0.9$ , and  $\sigma^2 = 1$ . Note that this means that  $R(0) = 5.4$ . If  $\bar{Y} = 40.2$  based on a sample of size  $n = 100$  and someone calculated a 95% confidence interval for  $\mu$  ignoring the autocorrelation in the data, what would they get? What would they get if they then used the fact that there is autocorrelation?
8. (10 points)
  - a. Show that a spectral density is symmetric about  $\omega = 0.5$ .
  - b. Show that  $\text{TOEPL}(1, \rho(1))$  for an MA(1) must be nonsingular.
9. (20 points) On the next page are the plots of the population correlogram and partial correlogram for 10 different processes. For each of the processes, tell me whether you think it is an MA, and AR, or an ARMA process. If an AR or MA, tell me what order it is.

Process	Type	Process	Type	Process	Type
A		E		H	
B		F		I	
C		G		J	
D					