

Name: \_\_\_\_\_

1. (10 points) If  $X$  is the result of applying a  $2K + 1$  moving average smoother to a white noise process having variance  $\sigma^2$ , what is the variance of  $X(t)$ ? What happens to this variance as  $K$  gets large?
2. (10 points) Derive the largest value that  $\rho(1)$  can be for an  $MA(1, \beta, \sigma^2)$  process.
3. (10 points) Let  $z(t) - \frac{3}{2}z(t-1) + \frac{1}{2}z(t-2) = 0$  for  $t \geq 2$  where  $z(0) = 2$  and  $z(1) = 3$ . Solve the difference equation. What happens to  $z(t)$  as  $t \rightarrow \infty$ ?
4. (10 points) For  $v = 0, \dots, 99$ , what is the sample autocorrelation coefficient  $\hat{\rho}(v)$  for a data set  $x$  of length 100 points whose first 50 points are equal to  $-1$  and last 50 points are equal to  $1$ ?
5. (21 points) What are the following?
  - a. Natural frequencies given a time series of length  $n$ .
  - b. The sample partial autocorrelation coefficient of lag 3 for a data set of length  $n$ .
  - c. The frequencies of peaks in the periodogram of a series having a nonsinusoidal cycle (plus noise) of period  $d$ .
  - d. The first difference of a random walk.
  - e. The 12th difference of a cosine (plus white noise) of period 12.
  - f. The appearance of a data set for which  $\hat{\rho}(1)$  is close to  $-1$  and all the rest of the  $\hat{\rho}(v)$ 's are close to zero.
  - g. The appearance of a data set for which  $\hat{\rho}(3)$  is close to  $1$  and all the rest of the  $\hat{\rho}(v)$ 's are close to zero.
6. (9 points) Draw rough pictures of the cumulative periodogram of each of the following data sets (make sure to label the horizontal and vertical axes):
  - a. Excess of low frequency.
  - b. Excess of high frequency.
  - c. Excess of both low and high frequency.

7. (10 points) Show that applying first differencing twice to a series which is a quadratic function of time will produce a series which is a constant.

8. (20 points) On the next page are plots of the log of ten spectral densities. On the page after that are plots of ten time series data sets, each having 100 points. Please fill in the following table:

Series	Spectral Density
A	
B	
C	
D	
E	
F	
G	
H	
I	
J	