STAT 604 Exam 1 Solutions **Fall, 1999 Dr. Newton**

GRADES:

```
9| 1 3 5 7 9
8| 0 0 0 0 0 1 1 2 5
7| 0 0
6| 2 7 8
5| 2 7
4|
3| 4 6
```

---

1. (15 pts) Write a Fortran subroutine to calculate $C = A - B$ where A, B, and C are double precision $n \times n$ matrices.

SOLUTION:

```
      subroutine mtsub(a,b,n,ndim,c)

      double precision a(ndim,n),b(ndim,n),c(ndim,n)
      integer n,ndim,i,j

      do 10 i=1,10
      do 10 j=1,10
 10   c(i,j)=a(i,j)-b(i,j)

      return
      end
```

---

2. (15 pts) What are x_1, x_2, and x_3 in Newton's method for finding the solution to $f(x) = x^2\cos(2\pi x) = 0$ using starting value $x_0 = 2$? What is $f(x_3)$?

SOLUTION:

We have

$$f'(x) = 2x\cos(2\pi x) - 2\pi x^2 \sin(2\pi x),$$

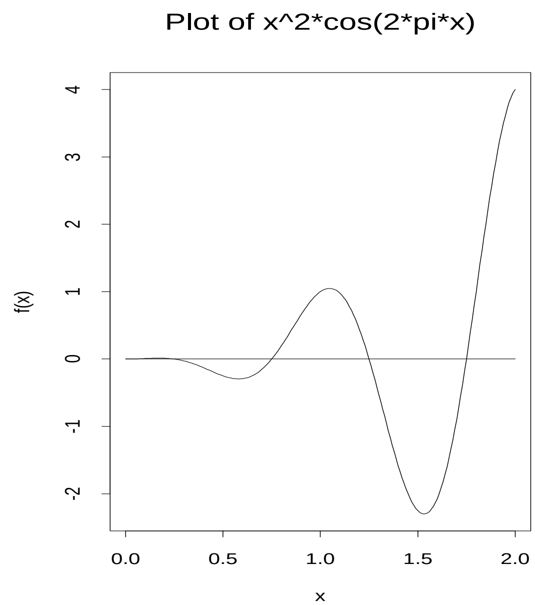
so the iterations are

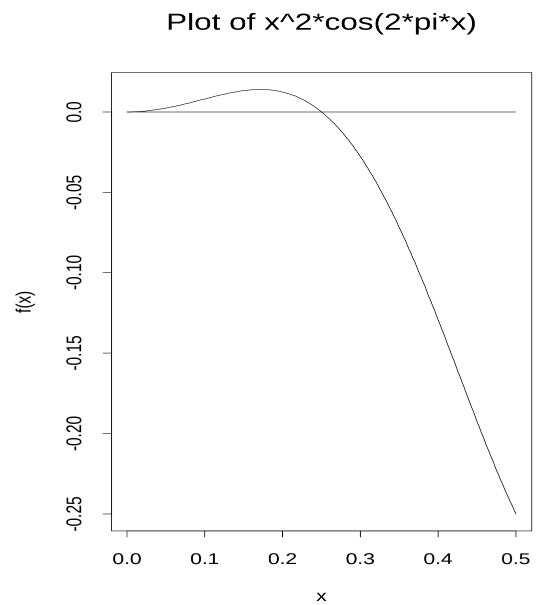
$$x_{i+1} = x_i - \delta_i,$$

where

$$\delta_i = \frac{f(x_i)}{f'(x_i)} = \frac{x_i^2\cos(2\pi x)}{2x_i\cos(2\pi x_i) - 2\pi x_i^2\sin(2\pi x_i)},$$

which gives $x_1 = 1$, $x_2 = 1/2$, $x_3 = 1/4$, and $f(x_3) = 0$, which means the process found a zero of f in three iterations.





---

3. (15 pts) What are the smallest, largest, and average values of the number of "update steps" in the algorithm we discussed for finding the maximum value of a vector when it is applied to random permutations of the integers 1 though 4?

SOLUTION:

The algorithm we discussed starts by setting the max as x_1 and then working through the rest of the x's and "updating" the max when we find an x greater than its current value. Thus the smallest number of updates is zero which happens anytime the first x is the overall max, while the largest number is three which happens when the x's are sorted from smallest to largest. To get the average value, one must determine the number of updates for each of the 24 permutations of 1 through 4, which are

```
1 2 3 4:  3    2 1 3 4:  2    3 1 2 4:  1    4 1 2 3:  0
1 2 4 3:  2    2 1 4 3:  1    3 1 4 2:  1    4 1 3 2:  0
1 3 2 4:  2    2 3 1 4:  2    3 2 1 4:  1    4 2 1 3:  0
1 3 4 2:  2    2 3 4 1:  2    3 2 4 1:  1    4 2 3 1:  0
1 4 2 3:  1    2 4 1 3:  1    3 4 1 2:  1    4 3 1 2:  0
1 4 3 2:  1    2 4 3 1:  1    3 4 2 1:  1    4 3 2 1:  0
```

which gives a total of 26 updates, so the average is 26/24.

---

4. (15 pts) How many reps would I have to use in a simulation study of a probability p to be sure that I will be 99% sure that I know p to within ± 0.01?

SOLUTION:

Since $Z_{0.005} = 2.576$, we are 99% sure that $\hat{p}$ is within $2.576\sqrt{p(1-p)/r}$ of p, which is maximized for $p = 0.5$, and so we need to solve $0.01 = 2.576\sqrt{1/4r}$ for r, which gives $r = 16590$.

---

5. (10 pts) For a simple linear regression model with zero intercept, that is, $y_i = \beta x_i + \epsilon_i$, for $i = 1, \ldots, n$, use the result about sweeping the upper left hand corner of the matrix

$$A = \begin{bmatrix} X^T X & X^T y \\ y^T X & y^T y \end{bmatrix}$$

to find a formula for RSS.

SOLUTION:

We know sweeping A on the diagonals of $X^T X$ gives for the lower right hand element $y^T y - y^T X (X^T X)^{-1} X^T y$. In this example, we have

$$y^T y = \sum y_i^2, \qquad X^T y = y^T X = \sum x_i y_i, \qquad (X^T X)^{-1} = 1/\sum x_i^2,$$

and so

$$RSS = \sum y_i^2 - \left(\sum x_i y_i\right)^2 / \sum x_i^2.$$

---

6. (10 pts) How is $x = -52.6875$ represented as a `real*4`?

SOLUTION:

Since $52.6875 = 2^5 + 2^4 + 2^2 + 2^{-1} + 2^{-3} + 2^{-4}$, we have

```
52.6875D=11010.1011B=1.10101011 x 10^5 B
```

so the first bit is 1 (because x is negative), the next 8 bits are the binary representation of $132 = 5 + 127$, and the last 23 bits are those to the right of the binary point (with 0's added to fill out the bits). Thus we have `1100 0010 0101 0010 1100 0000 0000 0000`.

---

7. (10 pts) If I pass a matrix A that is declared to be 10×10 in the main program to a subroutine which is expecting the matrix B that is declared to be 5×5 in the subroutine, which element of A will be treated as B_{55}?

SOLUTION:

The 100 elements of A are passed to the subroutine as a vector by stringing out its columns. The matrix B is formed by "unstringing out" these 100 elements by putting the first 5 in B's first column, the next 5 in its second column, and so on. Thus $B_{55} = A_{53}$.

---

8. (10 pts) If I let X be the number of defective items in a random sample (without replacement) of n items from a group of N items, D of which are defective, then

$$f(x) = P(X = x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}.$$

If I know $f(x)$ for some x, how can I find $f(x+1)$ without actually evaluating the above formula?

SOLUTION:

We have

$$\frac{f(x+1)}{f(x)} = \frac{(D-x)(n-x)}{(x+1)(N-D-n+x+1)} = g_x,$$

which means $f(x+1) = g_x f(x)$.