

Name: _____

1. (10 pts) How would you T_EX the following?

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$$

2. (10 pts) What would be the result of the first splitting of

19 13 1 2 12 16 18 4 14 9 5 15 8 7 17 20 10 6 11 3

in quicksort?

3. (10 pts) How many reps (Bernoulli trials) must one perform in a simulation in order to be at least 95% sure that one is estimating a population proportion within ± 0.01 ?
4. (10 pts) A megaflops is defined to be one million double precision floating point operations per second. On a two megaflops computer, how long does it take to multiply two 100 by 100 matrices 50 times?
5. (20 pts) Given a sample of size n from the lognormal distribution

$$f(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma x}} e^{(-\log x - \mu)^2 / 2\sigma^2}, \quad x > 0, \mu \in \mathcal{R}, \sigma > 0,$$

find the gradient and Hessian matrices to be used in minimizing via Newton–Raphson the negative of the log likelihood of μ and σ^2 .

6. (20 pts) How would you use the U(0,1) distribution in the rejection method to generate random numbers from the beta distribution having pdf

$$f(x) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}, \quad x \in (0, 1), \alpha > 0, \beta > 0,$$

for $\alpha = \beta$ for integer values of $\alpha > 1$? For a given value of α , what is the probability of rejection? What is this probability for $\alpha = 3$?

7. (20 pts) Write an Splus function `ptnorm(sig1, sig2)` that will superimpose plots of the $N(0, \sigma^2)$ pdf for 101 equally spaced values of σ in `[sig1, sig2]`, with each pdf plotted at 301 equally spaced points from the .001 to .999 quantile of the distribution. The function must call `plot` to set up the axes and then do a `for` loop that will call `lines` to draw each pdf.