STAT 604 Final Exam **Fall, 1997 Dr. Newton**

Name: ______________________________

1. (10 pts) The Chebychev polynomials are defined by

$$T_0(x) = 1, \qquad T_1(x) = x, \qquad T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x), \quad n > 1.$$

What is $T_4(x)$?

2. (10 pts) What is the square root of the matrix

$$A = \begin{bmatrix} 6 & 3 & 2 \\ 3 & 5 & 1 \\ 2 & 1 & 8 \end{bmatrix}$$

3. (10 pts) What is the estimate of σ^2 obtained by jackknifing s^2 for a sample of size n?

4. (20 pts) Which function has a sharper peak at $x = 0$, the standard normal pdf f or the standard Cauchy pdf

$$g(x) = \frac{1}{\pi(1+x^2)}?$$

5. (10 pts) What is the relative absolute error in approximating $\int_0^1 x^2 dx$ by the trapezoidal rule using 21 grid points.

6. (10 pts) If all you have is a N(0,1) random number generator, how would you generate χ^2_v and F_{v_1,v_2} random variables?

7. (10 pts) What is the result of making the vector $y = (1,2,3,4,5)^T$ orthogonal to the vector $x = (6,4,8,3,1)^T$?

8. (10 pts) How would you $\TeX$

$$g(\alpha, \beta) = n \log \beta + \frac{1}{2\beta^2} \sum_{i=1}^{n} (\log x_i - \alpha)^2?$$

9. (10 pts) What is the quantile function corresponding to the pdf

$$f(x) = \frac{1}{\alpha^2} x e^{-x^2/(2\alpha^2)}, \qquad x > 0?$$

10. (10 pts) If $A = \begin{bmatrix} B & C \\ D & E \end{bmatrix}$, where B and E are $(r \times r)$ and $(s \times s)$, respectively, show (by first sweeping A on its first r diagonals and the on the last s diagonals) that the upper left hand $(r \times r)$ part of A^{-1} is given by

$$F = B^{-1} + B^{-1}C(E - DB^{-1}C)^{-1}DB^{-1}.$$