STAT 604 Final Exam **Fall 1996 Dr. Newton**

Instructions: Do all problems on the test paper, using the back if you need more room for a particular problem.

1. (8 points) Why does the following Fortran code print `k = 0`?

```
      data k/2048/

      call prt(k)

      stop
      end

      subroutine prt(k)
      integer*2 k

      write(*,10) k
 10   format(' k = ',i5)

      return
      end
```

2. (10 points) Let $\tilde{X}_n$ be the median of $n = 2k+1$ distinct observations $X_1, \ldots, X_n$. Show that the jackknifed version of $\tilde{X}_n$ (the average of the n "leave one out" medians) is given by

$$\hat{X} = \frac{k}{n}\tilde{X}_n + \frac{k+1}{2n}\left[X_{(k)} + X_{(k+2)}\right],$$

where the order statisics of the data are denoted by $X_{(1)} < \cdots < X_{(n)}$.

3. (8 points) In quick sort, what is the result of the first splitting of the array

```
13 11 9 7 6 8 3 14 10 4 15 5 2 12 1?
```

4. (10 points) What are the quantile functions of:

a. The Rayleigh distribution

$$f(x) = \frac{1}{\alpha^2} x e^{-x^2/(2\alpha^2)}, \quad x > 0,$$

b. The Weibull distribution

$$f(x) = \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}, \quad x > 0.$$

5. (8 points) Find the two errors in the following TEX code for typing the formula for the Weibull pdf.

```
$$
f(x)=\alpha\betax^{\beta-1}e^{-\alpha x^{\beta},\quad x>0.
$$
```

6. (8 points) Use the definition of positive definiteness to prove that the following matrix is not positive definite.

$$A = \begin{bmatrix} 10 & 5 & 3 \\ 5 & 5 & 6 \\ 3 & 6 & -1 \end{bmatrix}?$$

7. (8 points) Show that the determinant of a positive definite matrix is the product of the diagonal elements of the diagonal matrix in its MCD.

8. (8 points) Suppose that we are confronted with a computing task of size n (such as sorting a set of n numbers or multiplying two $n \times n$ matrices) that requires $f(n)$ operations (for example, multiplying two $n \times n$ matrices takes $f(n) = n^3$ multiplications and additions). Suppose that we can perform the entire task by first dividing the problem into smaller versions of the same task and then combining the solutions of the smaller tasks into the solution of the larger task. Then the total number of operations required may be greatly reduced relative to $f(n)$. Suppose we can divide the problem in half and n is a power of 2, and the number of operations to combine the solution of the small problems into the solution to the big problem is negligible, what is the total number of operations if we continue dividing the problem in half until we are left with problems of size 2?

9. (8 points) Why do we convert optimization problems into minimization problems when using Newton-Raphson?

10. (8 points) How do we use quick sort to find the ranks of a vector?

11. (8 points) What are the `external` and `common` statements used for in Fortran?

12. (8 points) Write a single line of Splus code that will generate a vector of n U(0,1)'s and count how many of them are less than or equal to a number p that is between 0 and 1. What is the probability distribution of the resulting random variable?