STAT 604 Exam 2 **Fall, 1996 Dr. Newton**

Instructions: Do all problems on the test paper, using the back if you need more room for a particular problem.

1. (15 points) Write down an algorithm that uses the standard Cauchy pdf

$$f(x) = \frac{1}{\pi}\frac{1}{1+x^2}, \qquad -\infty < x < \infty,$$

with the rejection method to generate N(0,1) observations. Be sure to choose the constant c in an optimal way and to describe how to generate observations from the Cauchy.

2. (10 points) Find the method of moments estimators of μ and σ^2 from a $\mathrm{N}(\mu, \sigma^2)$ sample.

3. (10 points) Let $X \sim \mathrm{Bin}(n, p)$ and let $f_{n,k} = P(X = k)$. Find two recursions for $f_{n,k}$, one in k and one in n.

4. (15 points) Wite a Fortran subroutine that uses `runif(dseed,n,u)` to generate a random vector $(X_1, \ldots, X_K)$ from a multinomial distribution with parameters n and $p_1, \ldots, p_K$, where the p's are inputted as integers summing to 1000.

5. (15 points) Write an S Plus function that will generate a $(2 \times n)$ matrix X whose columns are iid bivariate normal random vectors with mean zero and covariance matrix having ones on the diagonal and user specified value ρ on the off diagonals.

6. (5 points) How much memory is needed to store (without compression) a region on a computer screen that is (1024×1024) where each picel can be as many as 256 colors?

7. (5 points) In an eight by eight pixel region on the screen, what letter of the alphabet is represented by the eight unsigned integers 240, 96, 96, 96, 98, 102, 254, 0?

8. (15 points) Assuming you have starting values, how would you use Newton-Raphson to minimize with respect to β_1 and β_2 the function

$$S(\beta_1, \beta_2) = \sum_{i=1}^{n} \left(y_i - \beta_1 e^{\beta_2 X_i}\right)^2 .$$

9. (10 points) How would you $\TeX$ the displayed equation in problem 8?