STAT 604 Final Exam **Fall, 1995 Dr. Newton**

1. (12 points) In a regression problem where X is $(n \times m)$, show that the vector $\hat{y}$ of 'fitted values' can be written as $\hat{y} = Hy$, for some $(n \times n)$ matrix H and find H. Using the fact that $\mathrm{tr}(AB) = \mathrm{tr}(BA)$, show that the average value of the diagonal elements of H is m/n (the trace of a square matrix is the sum of its diagonal elements). Express H_{ii} as a function of the elements of the matrices you get from the Modified Gram Schmidt Decomposition of X.

2. (8 points) Show that a kernel density estimator is a pdf.

3. (10 points) Find a simple recursion for the log of Poisson probabilities $f(x)$ for $x \geq 0$ and use this to write a fortran subroutine that will return $f(x)$ for x up to M for user specified M and Poisson mean value parameter λ.

4. (12 points) For a nonlinear regression problem having mean function f, express the gradient and Hessian in terms of the Jacobian, residual vector, and the matrices of second derivatives of f with respect to parameters. What is the difference between Newton-Raphson and Gauss-Newton in this situation?

5. (12 points) If from a regression problem all I wanted were least squares estimators for coefficients, which method has more numerical operations, the MCD or the MGS?

6. (8 points) What is the relative absolute error in using the composite Simpson's rule (with four intervals) to approximate $\int_0^1 \cos \pi x dx$?

7. (8 points) In constructing a histogram of data $X_1, \ldots, X_n$, how can I find the frequencies without using any if statements?

8. (10 points) Why does the program below print

```
 n =          -2
```

```
      n=-123456
      call prtsc(n)
      stop
      end

      subroutine prtsc(n)
      integer*2 n
      write(*,10) n
 10   format(' n = ',i10)
      return
      end
```

9. (10 points) To generate n pairs of iid N(0,1) random variables using the usual normal random number generator, how many pairs of U(0,1)'s do you need to generate to be 95% sure that you have enough to generate the normals?

10. (10 points) In quick sort, what would be the result of the first splitting of the array

4 13 11 6 2 10 5 1 3 8 7 12 9?