1. (20 points) There are many formulas for SSR, but the definition is $SSR = \sum_{i=1}^{n} e_i^2$, where the vector $e = y - X\hat{\beta}$ (and so $e_i = y_i - i$th row of $X$ times $\hat{\beta}$). Thus we have

```c
double precision function ssr(y,X,betah,n,m,ndim)
  double precision y(n),X(ndim,m),betah(m),ei
  ssr=0.0d0
  do 20 i=1,n
    c Get e_i:
    ei=y(i)
    do 10 j=1,m
      10 ei=ei-betah(j)*X(i,j)
  20 ssr=ssr+ei*ei
  return
end
```

2. (10 points) The basic point is that $(LU)_{i,j} = \sum_{k=1}^{n} L_{ik}U_{kj} = \sum_{k=1}^{\text{min}(i,j)} L_{ik}U_{kj}$, since $L_{ik} = 0$ for $k > i$ and $U_{kj} = 0$ for $k > j$. Thus the number of multiplications and additions for the $(i,j)$th element of the product is $\text{min}(i,j)$ (in counting operations in an inner product, it is standard to ignore the fact that there is one less addition than multiplications) and thus the total number is

$$T_n = \sum_{i=1}^{n} \sum_{j=1}^{n} \text{min}(i,j).$$

If we break the sum over $i$ into 1 to $n - 1$ and $n$ by itself, and then the sum over $j$ into 1 to $i$ and $i + 1$ to $n$, we get

$$T_n = \sum_{i=1}^{n-1} \left[ \sum_{j=1}^{i} \text{min}(i,j) + \sum_{j=i+1}^{n} \text{min}(i,j) \right] + \sum_{j=1}^{n} \text{min}(n,j)$$

$$= \sum_{i=1}^{n-1} \left[ \sum_{j=1}^{i} j + \sum_{j=i+1}^{n} i \right] + \sum_{j=1}^{n} j,$$

which after lots of algebra (and using the facts that the sum and sum of squares of the first $n$ positive integers are $n(n+1)/2$ and $n(n+1)(2n+1)/6$) ends up being

$$T_n = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6} \sim \frac{n^3}{3},$$

as opposed to $n^3$ if we just multiply two $(n \times n)$ matrices. (Note that the notation $a_n \sim b_n$ means $\lim_{n \to \infty} a_n/b_n \to 1$.)

There is actually a very simple way to determine $T_n$. It is the sum of the elements of a matrix like the one for $n = 4$:
This matrix for \( n \) is the same as the one for \( n - 1 \) with an extra row and column appended and the sum of elements in the new row and column is \( 2(n - 1)n/2 + n = n^2 \). Thus \( T_n = T_{n-1} + n^2 \), and since \( T_1 = 1 \), we have \( T_n \) is the sum of squares of the first \( n \) positive integers which is \( n(n + 1)(2n + 1)/6 \).

3. (10 points) We have

\[
\begin{align*}
735 &= 512 + 128 + 64 + 16 + 8 + 4 + 2 + 1 = 0000\ 0010\ 1101\ 1111 \\
537 &= 512 + 16 + 8 = 1.000011001 \times 2^{-9} = 0\ 10001000\ 000011001 \quad 14 \text{ 0’s} \\
\text{since} \ 9 + 127 &= 136 = 128 + 8 = 10001000 \\
\text{Thus} \ -537 &= 1100\ 0100\ 0000\ 0110\ 0100\ 0000\ 0000\ 0000
\end{align*}
\]

4. (15 points) Since there is no \texttt{ndim} in the subroutine, the first 20 elements of \texttt{x} (which is its first column) will be reshaped into a 10 by 2 matrix whose first column are 1’s and second column 0’s, and thus

\[
\begin{array}{cccccccccc}
\text{x in main} & \text{x in subroutine} & \text{xtx in sub} & \text{xtx in main} \\
\hline
1 & 1 & 10 & 0 & 10 & 0 & 0 & 0 & 0 \\
1 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 6 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 7 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 10 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

what’s written:

\[
\begin{align*}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
. & . \\
. & . \\
0 & 0
\end{align*}
\]
5. (20 points) If we let \( B = \text{SWEEP}(k)A \) and \( C = \text{SWEEP}(k)B \), then using the definition of SWEEP, it is easy to show for \( i \neq k \) and \( j \neq k \), that \( C_{kk} = A_{kk} \), \( C_{ik} = A_{ik} \), \( C_{kj} = A_{kj} \), and \( C_{ij} = A_{ij} \).

6. (10 points) We have \( \chi^2_m = \sum_{i=1}^{m} Z_i^2 \), where \( Z_1, \ldots, Z_m \) are iid \( \text{N}(0,1) \) while

\[
F_{m_1, m_2} = \frac{\chi^2_{m_1}/m_1}{\chi^2_{m_2}/m_2},
\]

where the numerator and denominator of \( F_{m_1, m_2} \) are independent. Thus all we need is a \( \text{N}(0,1) \) random number generator to get \( \chi^2 \)'s and \( F \)'s (there are faster ways to do these but this would work).

7. (15 points) By random point, we mean that the probability that the point falls in a certain region is proportional to the area of that region, and thus the chance that a point in the square also is in the circle is the ratio of the two areas, namely \( p = \pi/4 \). Thus if we generate \( n \) points in the square (by generating two \( U(-1, 1)'s \) and let \( \hat{p} = X/n \) where \( X \) is the number of points in the circle, we can get an estimate of \( \pi \) as \( 4\hat{p} \). Thus to have \( \pi \) fall in an interval of width .01, we need \( p \) to fall in an interval of width .01/4. Thus from the large sample binomial confidence interval for \( p \) we need

\[
\frac{.01}{4} = \frac{1.96(2) \sqrt{p(1-p)}}{\sqrt{n}} \leq \frac{1.96}{\sqrt{n}},
\]

since \( p(1 - p) \leq 1/4 \). Solving this gives \( n = 614,656 \).