STAT 604 Final Exam **Fall, 1994 Dr. Newton**

1. (10 points) Given the `real*4 function runif(dseed)` that generates a single $U(0,1)$ for input and output double precision seed `dseed`, write `subroutine rlogist(dseed,n,alpha,beta,x)` which generates n observations $X_1, \ldots, X_n$ from the logistic distribution

$$f(x;\mu,\beta) = \frac{1}{\beta}\frac{e^{-(x-\alpha)/\beta}}{[1+e^{-(x-\alpha)/\beta}]^2}, \qquad x \in \mathcal{R}.$$

2. (15 points) For the gamma density having parameter $\nu > 1$

$$f_\Gamma(x) = \frac{x^{\nu-1}e^{-x}}{\Gamma(\nu)}, \qquad x > 0,$$

find a double exponential pdf

$$f_{DE}(x;\mu,\sigma) = \frac{1}{2\sigma}e^{-|x-\mu|/\sigma}, \qquad x \in \mathcal{R},\ \mu \in \mathcal{R},\ \sigma > 0$$

having the same mode as f_Γ and a constant c so that $h(x) = cf_{DE}(x) \geq f_\Gamma(x)$ for all x and c is as small as possible. How would you use this to generate gamma data using rejection?

3. (15 points) Let $\tilde{X}_n$ be the median of $n = 2k+1$ distinct observations $X_1, \ldots, X_n$. Show that the jackknifed version of $\tilde{X}_n$ (the average of the n "leave one out" medians) is given by

$$\hat{X} = \frac{k}{n}\tilde{X}_n + \frac{k+1}{2n}\left[X_{(k)} + X_{(k+2)}\right],$$

where the order statisics of the data are denoted by $X_{(1)} < \cdots < X_{(n)}$.

4. (15 points) Find a recursive method for finding the sample variance s_n^2 of a sample $X_1, \ldots, X_n$, that is, a formula which finds s_n^2 as a function only of s_{n-1}^2, $\bar{X}_{n-1}$, and X_n.

5. (15 points) Let X be a realization from a hypergeometric distribution with parameters N, D, and n, that is, X is the number of defectives in a random sample (without replacement) of n objects from a population of size N objects which contains D defectives. Write down the probability distribution of X and express $f(x+1)$ as a function of $f(x)$ and the parameters of the distribution.

6. (15 points) Let X be an $(n \times m)$ matrix and let $A = X^T X$.

a. How do we check whether X is of full rank?

b. How do we check whether A is positive definite if we don't know X?

c. How do we find the square root of A?

d. How do we find the determinant of A?

e. How do we solve a system of equations $Ax = b$?

7. (15 points) How would you modify the idea of Bresenham's line drawing method to draw a circle centered at pixel (c_0, r_0) of radius r pixels.