

1. (25 points) If I let  $A_n$  denote the  $(n \times n)$  matrix whose  $(i, j)$ th element is  $10i + j$ , what is the trace of  $A_3$ , that is, the sum of the diagonal elements of  $A_3$ ? What would you get from the following Fortran program?

```

      dimension a(10,10)
      real*4 trace

      do 10 i=1,10
      do 10 j=1,10
10    a(i,j)=10*i+j

      write(*,*) trace(a,3)

      stop
      end
      real*4 function trace(a,n)
      dimension a(n,n)

      trace=0.0
      do 10 i=1,n
10    trace=trace+a(i,i)

      return
      end

```

2. (25 points) What is

$$\text{SWEEP}(2) \begin{bmatrix} 6 & 4 & 3 \\ 4 & 5 & 3 \\ 3 & 3 & 4 \end{bmatrix} ?$$

3. (25 points) Express  $-305$  as an `integer*2` and  $473.1250$  as a `real*4`.

4. (25 points) In a regression problem  $y = X\beta + \epsilon$ , where  $X$  is  $(n \times m)$ , the matrix  $H = X(X^T X)^{-1} X^T$  is called the “hat matrix” since multiplying  $y$  by  $H$  gives  $\hat{y}$ . The diagonal elements of  $H$  play an important role in analyzing a regression problem for outlying or influential points. Show that

$$H_{ii} = \sum_{j=1}^m Q_{ij}^2 / D_{jj}, \quad i = 1, \dots, n,$$

where  $D = Q^T Q$  and  $X = QR$  is the Gram Schmidt decomposition of  $X$ .