

1. (15 points) Show that the jackknife estimator of the standard error of \bar{X} is the same as the usual estimated standard error.

2. (15 points) Prove that

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^T A^{-1}}{1 + v^T A^{-1}u},$$

where A is $(n \times n)$ and u and v are $(n \times 1)$.

3. (15 points) How would you use Newton-Raphson to find a relative minimum of the function

$$S(\theta) = (2\theta_1^2 + 3\theta_2^2) \exp(-\theta_1^2 - \theta_2^2)?$$

Assume you have a reasonable starting value for the process.

4. (15 points) Consider a computer screen having n columns and m rows of pixels where the lower left hand corner pixel is numbered $(0, 0)$ (column 0, row 0). Suppose we want to use the rectangular region of the screen having lower left corner pixel (c_1, r_1) and upper right corner pixel (c_2, r_2) to represent a “real world” region having lower left corner (x_1, y_1) and upper right corner (x_2, y_2) . Which pixel corresponds to a real world coordinate (x, y) ?

5. (10 points) Show that a kernel density estimator integrates to one.

6. (15 points) Write a Fortran subroutine that multiplies an $(n \times n)$ unit lower triangular matrix L times a $(n \times n)$ unit upper triangular matrix R that never multiplies by the elements of L or R that we know are 1 or 0.

7. (15 points) What would be the new values in the array $(6,4,10,2,3,9,5,8,7,11,1)$ after the first splitting in the quicksort algorithm?