STAT 604 Exam 1 **Fall, 1992 Dr. Newton**

1. (10 points) What is the square root of the matrix

$$A = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$$

2. (10 points) Prove that a kernel density estimate is in fact a probability density function.

3. (20 points) Write a self-contained Fortran *function* that will calculate the quadratic form $x^T Ax$, where x is a vector of length n and A is an $(n \times n)$ matrix. How many operations does it take to do this?

4. (20 points) If the lower left hand corner of a computer screen corresponds to values (x_{min}, y_{min}) of x and y, while the upper right hand corner is (x_{max}, y_{max}), and the screen has n_x and n_y columns and rows of pixels, in which column and row of pixels would the value (x_0, y_0) of (x, y) belong? Note that the columns and rows are numbered from 0 to $n_x - 1$ and 0 to $n_y - 1$, respectively.

5. (20 points) How would you TEX the following?

Let $X_1, \ldots, X_n$ be i.i.d. with density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{\sigma^2}}, \qquad -\infty < x < \infty.$$

6. (20 points) If you have a U(0,1) random number generator, how would you generate a random sample of size n from the Weibull distribution, that is, the distribution having pdf

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}, & x > 0, \alpha > 0, \beta > 0 \\ 0, & \text{otherwise.} \end{cases}$$