

1. (10 points) What is the square root of the matrix

$$A = \begin{bmatrix} 10 & 6 \\ 6 & 10 \end{bmatrix}$$

2. (10 points) Prove that a kernel density estimate is in fact a probability density function.
3. (20 points) Write a self-contained Fortran *function* that will calculate the quadratic form  $x^T Ax$ , where  $x$  is a vector of length  $n$  and  $A$  is an  $(n \times n)$  matrix. How many operations does it take to do this?
4. (20 points) If the lower left hand corner of a computer screen corresponds to values  $(x_{min}, y_{min})$  of  $x$  and  $y$ , while the upper right hand corner is  $(x_{max}, y_{max})$ , and the screen has  $n_x$  and  $n_y$  columns and rows of pixels, in which column and row of pixels would the value  $(x_0, y_0)$  of  $(x, y)$  belong? Note that the columns and rows are numbered from 0 to  $n_x - 1$  and 0 to  $n_y - 1$ , respectively.
5. (20 points) How would you  $\text{\TeX}$  the following?

Let  $X_1, \dots, X_n$  be i.i.d. with density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{\sigma^2}}, \quad -\infty < x < \infty.$$

6. (20 points) If you have a  $U(0,1)$  random number generator, how would you generate a random sample of size  $n$  from the Weibull distribution, that is, the distribution having pdf

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1} e^{-\alpha x^\beta}, & x > 0, \alpha > 0, \beta > 0 \\ 0, & \text{otherwise.} \end{cases}$$