STAT 604 Final Exam **Fall, 1991 Dr. Newton**

Name: _____

1. (10 points) How is -123.983 represented as a `real*4`? How about 1,532 as an `integer*2`?

2. (10 points) How would you use Newton-Raphson to find a relative minimum of the function

$$S(\theta) = (\theta_1 + 10\theta_2)^2 + 5(\theta_3 - \theta_4)^2 + (\theta_2 - 2\theta_3)^4 + 10(\theta_1 - \theta_4)^4?$$

Assume you have a reasonable starting value for the process.

3. (10 points) What is wrong with the following TEX code (in order to generate the displayed equation in the previous question)?

```
$$
S(\theta)=(\theta_1+10\theta_2)**2+5(theta_3-\theta_4)^2
  +(\theta_2-2\theta_3)^4+10(\theta_1-\theta_4)^4?
$
```

4. (10 points) If you knew that a vector $X(1), \ldots, X(n)$ was already arranged in descending order, which would you use to arrange the vector into ascending order, quicksort or bubble sort? Why?

5. (10 points) How many multiplications and additions are required to invert a unit lower triangular $n \times n$ matrix using the efficient algorithm we discussed in class? How many are needed if we just invert it using the sweep algorithm (without being at all clever)?

6. (10 points) What does it mean to say that the permutation $(i_1, \ldots, i_n)$ of the integers $1, \ldots, n$ is random? Show that when $n = 2$, the algorithm we described in class for finding random permutations does indeed produce random permutations.

7. (10 points) Show that the Chebychev polynomials

$$T_0 = 1, \quad T_1(x) = x, \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n \geq 1,$$

are orthogonal with respect to $w(x) = (1 - x^2)^{-1/2}$ on $[-1, 1]$.

8. (10 points) Prove that

$$(A + uv^T)^{-1} = A^{-1} - \frac{A^{-1}uv^TA^{-1}}{1 + v^TA^{-1}u},$$

where A is $(n \times n)$ and u and v are $(n \times 1)$.

9. (10 points) Define the inverse square root $V^{-1/2}$ of the $(n \times n)$ positive definite matrix V. Now suppose that $y = X\beta + \epsilon$ is an ordinary, fixed-X, full rank, $(n \times p)$ regression model except that the variance-covariance matrix of ϵ is V instead of the usual $\sigma^2 I$. If we multiply all three terms in the model by $V^{-1/2}$, what would be the resulting normal equations?

10. (10 points) Show that the jackknife standard error of $\bar{X}$ is the same as the usual estimated standard error.