STAT 604 Exam 2 Fall, 1991 Dr. Newton

1. (15 points) Suppose I have data $X_1, \ldots, X_n$ which fall between a and b and I divide $[a, b]$ into m intervals of equal length. Write a subroutine that, in one pass through the data, and without actually calculating the endpoints of the intervals, will return an array `ii(1),...,ii(m)` containing the number of X's in each of the m intervals.

2. (15 points) The Rayleigh distribution with parameter $\alpha > 0$ has pdf

$$f(x) = \frac{1}{\alpha^2} x e^{-x^2/(2\alpha^2)}, \qquad x > 0.$$

a. What is the cdf F of this distribution?
b. Thus what is its quantile function?
c. Thus, given a U(0,1) random number generator, how would you simulate data $X_1, \ldots, X_n$ from this distribution?

3. (15 points) How would you estimate the standard error of the interquartile range of a population having pdf f given a random sample $X_1, \ldots, X_n$ from the population? If f is the exponential density with parameter $\lambda = 1$, what is the true interquartile range?

4. (15 points) Given data $y_1, \ldots, y_n$ from a distribution having density f, and given abscissas $x_1, \ldots, x_m$:

a. Define the kernel density estimator $\hat{f}_{K,h}$ of f for kernel K and bandwidth h.
b. Define two kernels that might be used.
c. What happens to the smoothness of $\hat{f}_{K,h}$ for a fixed data set and kernel as h increases? Why?

5. (10 points) What would be printed as a result of the following Fortran code?

```
        dimension x(7,6)
        do 10 i=1,7
        do 10 j=1,6
10      x(i,j)=10*(i-1)+j
        call mprt(x,3,3)
        call mprt(x,5,5)
        stop
        end
        subroutine mprt(a,n,m)
        dimension a(n,m)
        do 10 i=1,n
10      write(*,*) (a(i,j),j=1,m)
        return
        end
```

6. (15 points) Write down in matrix form the multiple linear regression model, making sure to define the vectors and matrices involved and to specify their dimensions. Give the formula for the least squares estimators of the coefficients of the model, and those of the residual sum of squares and covariance matrix of the least squares estimators. How does one use the Modified Cholesky Decomposition to obtain these three quantities?

7. (15 points) Given a sample $X_1, \ldots, X_n$ from a population having mean μ, show that the jackknife estimator of μ is actually $\bar{X}$.