STAT 604 Final Exam **Fall, 1990 Dr. Newton**

Name: ____________________________

1. (12 points) How is -73.4236 represented as a `real*4`? How about 7,532 as an `integer*2`?

2. (12 points) How would you use Newton-Raphson to find a relative minimum of the function

$$S(\theta)=\sum_{i=1}^n(y_i-\theta_1X_{i1}-e^{\theta_2X_{i2}})^2,$$

where $\theta^T=(\theta_1,\theta_2)$ and we observe (y_i,X_{i1},X_{i2}) for $i=1,\ldots,n$. Assume you have a reasonable starting value for the process.

3. (8 points) What is wrong with the following TEX code (in order to generate the displayed equation in the previous question)?

```
$$
S(\theta)=\sum_{i=1}^n(y_i-\theta_1X_{i1}-e^{\theta_2X_{i2})^2,
$$
```

4. (10 points) Write down a recursive algorithm for finding $s_n^2=\sum_{i=1}^n(X_i-\bar{X})^2/(n-1)$ in terms of s_{n-1}^2 and X_n.

5. (8 points) What would be the output of the following Fortran code?

```
      dimension x(7,6)
      do 10 i=1,7
      do 10 j=1,6
10    x(i,j)=10*(i-1)+j
      call mprt(x,5,5)
      stop
      end
      subroutine mprt(a,n,m)
      dimension a(n,m)
      write(*,*) a(n,m)
      return
      end
```

6. (12 points) Write down in matrix form the multiple linear regression model, making sure to define the vectors and matrices involved and to specify their dimensions. Give the formula for the least squares estimators of the coefficients of the model, and those of the residual sum of squares and covariance matrix of the least squares estimators. How does one use the Modified Cholesky Decomposition to obtain these three quantities?

7. (14 points) Write an S function (call it `plotdray`) that will plot the pdf of the Rayleigh distribution

$$f(x) = \frac{1}{\alpha^2} x e^{-x^2/(2\alpha^2)}, \qquad x > 0,$$

from its `u1`th to `u2`th quantiles. Write functions `dray` and `qray` to use in the `plotdray` function. The arguments to `plotdray` should be `alpha`, `u1`, and `u2` which should be assigned default values 1.0, 0.01, and 0.99, respectively. You can assume that the user has already called `x11` or `postscript` prior to calling `plotdray`. Nice labels on the plot would be useful but not required.

8. (12 points) What is the relative absolute error in using the composite Simpson's rule (with four intervals) to approximate $\int_0^1 \cos \pi x \, dx$?

9. (12 points) What is the probability that a random point in the unit square is also in the unit circle? Use this to design a simulation that estimates π by generating random points in the unit square. How many points would you have to generate to be 95% sure that your estimate of π is within 0.01 of the true value?