Outline of Lecture 11, STAT 303, Sections 503–4

Homework (Due Friday 2/16)

1. *Bivariate descriptive statistics concept lab*, Questions 2, 3, 4 in notes.
2. *Probability:* Problems 4.4, 8, 10, 12, 22
3. *Random variables:* Problems 4.34, 36, 44, 58, 72
4. *Binomial probabilities:* Problems 5.2, 4, 8, 12 (use `Calculator -> Statistical tables -> Binomial` for Problems 4, 8, and 12).

Exam 1

1. Because of Martin Luther King Holiday and the bad weather day, postpone exam until Friday February 23.
2. Cover material through Wednesday February 14.

Four Discrete Random Variables

1. Binomial, Hypergeometric, Negative Binomial, Poisson random variables.
2. Before defining, need two ideas:
   a. *0–1 population*, that is, population of items of two types, one labeled a 1 and the other labeled a 0 (a 1 is often called a *success* and a 0 a *failure*). Let N be the number of elements in the population and let M be the number of elements that are a 1 and let π denote the proportion of 1's, that is, $\pi = M/N$.
   b. *Sampling with and without replacement.* Sampling with replacement means randomly draw an element and record whether it is a 0 or 1, then put the element back, then draw again, and so on (an element can be drawn more than once). Sampling without replacement is the same except we don't replace items once they are drawn.
3. Definition of Binomial, Hypergeometric, Negative Binomial, and Poisson random variables:
   a. $\text{Bin}(n, \pi)$: X is the number of 1's in a sample of size n (with replacement) from a 0–1 population. Note that this is equivalent to perfoming an experiment which can only be a success or a failure n times where π is the chance of a success on each experiment.
   b. $\text{Hyp}(N, M, n)$: Same as binomial except sampling without replacement.
   c. $\text{NBin}(r, \pi)$: X is the number of 0's drawn (with replacement) before getting the rth 1.
   d. $\text{Poiss}(\lambda)$ is same as binomial but is used when n is large and π is small. We let $\lambda = n\pi$.

What do we Want to Know About these Random Variables?

1. Their *probability distribution*, that is, a list of possible values of X and what each one's prob-

ability is. For example, for $\text{Bin}(n, \pi)$, possible values are $0, 1, \ldots, n$.

2. The *mean* of the random variable (also called *expected value*), that is, what would be the average value of X if we did the experiment many times. We use the notation μ_X or $\text{E}(X)$ to represent the mean of a random variable.

3. The *variance* of the random variable, that is, a measure of how much variability in the values of X if we did the experiment many times. We use the notation σ_X^2 or $\text{Var}(X)$ to denote the variance of a random variable.

Distribution, Mean, and Variance of Four Discrete Random Variables

Random Variable	$\Pr(X = x)$	μ_X	σ_X^2
$\text{Bin}(n, \pi)$	$\binom{n}{x}\pi^x(1-\pi)^{n-x}$, $x = 0, \ldots, n$	$n\pi$	$n\pi(1-\pi)$
$\text{Hyp}(N, M, n)$	$\dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$, $\max(0, n-(N-M)) \le x \le \min(n, M)$	$n\pi$	$\dfrac{N-n}{N-1}n\pi(1-\pi)$
$\text{NBin}(r, \pi)$	$\binom{x+r-1}{r-1}\pi^r(1-\pi)^x$, $x = 0, 1, 2, \ldots$	$\dfrac{r(1-\pi)}{\pi}$	$\dfrac{r(1-\pi)}{\pi^2}$
$\text{Poiss}(\lambda)$	$\dfrac{\lambda^x e^{-\lambda}}{x!}$, $x = 0, 1, 2, \ldots$	λ	λ

Note that $e = 2.7182818...$

Examples