More on Stratified Sampling

Suppose we wish to estimate the population mean $\mu$. First of all, how would we do so using the info from a stratified sample?

Let $\bar{y}_1, \ldots, \bar{y}_L$ be the sample means for the $L$ strata. The appropriate estimator of $\mu$ is

$$
\bar{y}_{\text{strata}} = \left( \frac{n_1}{n} \right) \bar{y}_1 + \cdots + \left( \frac{n_L}{n} \right) \bar{y}_L,
$$

where we assume that $n_i/n = N_i/N$ for each strata.

What is the variance of this estimator? Ignoring finite population corrections, the variance is

$$
n^{-1} \left[ \left( \frac{n_1}{n} \right) \sigma_1^2 + \cdots + \left( \frac{n_L}{n} \right) \sigma_L^2 \right],
$$

where $\sigma_i^2$ is the variance within the $i$th stratum.
For illustration’s sake, suppose the strata variances are all the same, say $\sigma^2_W$, where the $W$ stands for “within.”

In this case the variance of $\bar{y}_{\text{strata}}$ is $\sigma^2_W/n$.

What is the variance of $\bar{y}$ in a SRS of size $n$? We know the variance is $\sigma^2/n$, but how does this compare with $\sigma^2_W/n$?

When all the strata variances are equal to $\sigma^2_W$, then

$$\sigma^2 = \sigma^2_W + \left[ \frac{N_1}{N} (\mu_1 - \mu)^2 + \cdots + \frac{N_L}{N} (\mu_L - \mu)^2 \right].$$

So, stratified sampling will result in reduced variance in estimating the population mean unless all the strata means are the same.
Potentially, there is a huge reduction in variance. Suppose $\sigma_W$ is 0. Then we will estimate the population mean exactly with stratified sampling, but the variance of $\bar{y}$ for a SRS will be

$$\frac{1}{n} \left[ \left( \frac{N_1}{N} \right) (\mu_1 - \mu)^2 + \cdots + \left( \frac{N_L}{N} \right) (\mu_L - \mu)^2 \right].$$

Stratified sampling and blocking are similar in the following way:

Both strata and blocks should be chosen with the aim of minimizing within strata (or blocks) variation and maximizing between strata (or blocks) variation.