1. PPFPO (or poly) is a viscous liquid used extensively in the electronics industry as a lubricant. The infrared reflectance spectra properties of poly were examined. The optical density ($y$) for the infrared absorption of poly was recorded for different settings of band frequency ($x_1$) and film thickness ($x_2$).

(a) (8) Residuals were obtained after estimating the following model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon.$$

Discuss the adequacy of this model on the basis of the residual plots given on p. 4 of this exam.

A second model was fitted to these data using SPSS. The accompanying output contains an analysis for both the first and second models. In both models, $y$ is called $optical$, $x_1$ is called $band$ and $x_2$ $film$. In the second model, $X3 = x_1^2$, $X4 = x_2^2$, and $X5 = x_1 x_2$.

(b) (14) Carry out a hypothesis test to determine whether the second model significantly improves on the first one. Use $\alpha = .05$. 
Use the second model to answer the remaining parts of this question.

(c) (14) It was found that the standard error of \( \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_1^2 + \hat{\beta}_4 x_2^2 + \hat{\beta}_5 x_1 x_2 \)
when \( x_1 = 1 \) and \( x_2 = 1 \) is 0.067. Obtain a 95\% prediction interval for a single optical
density when \( x_1 = 1 \) and \( x_2 = 1 \).

(d) (8) What is the estimated variance of an error term?

(e) (14) Construct a 95\% confidence interval for \( \beta_1 \), the coefficient of band frequency.

(f) (14) Carry out a test to see if the term \( x_2^2 \) is useful in the model. Use \( \alpha = 0.10 \).
2. An experiment was conducted to determine the relationship between the amount of warping $Y$ for a particular alloy and the temperature (in °C) under which the experiment was conducted. Three observations were taken at each of 8 temperature settings.

(a) (14) Polynomial models of orders one through four were fitted to these data, and the following information was obtained:

<table>
<thead>
<tr>
<th>Order of Model</th>
<th>$R^2$</th>
<th>MSE</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9315</td>
<td>4.747</td>
<td>0.9284</td>
</tr>
<tr>
<td>2</td>
<td>0.9334</td>
<td>4.836</td>
<td>0.9271</td>
</tr>
<tr>
<td>3</td>
<td>0.9620</td>
<td>2.894</td>
<td>0.9563</td>
</tr>
<tr>
<td>4</td>
<td>0.9622</td>
<td>3.030</td>
<td>0.9542</td>
</tr>
</tbody>
</table>

$SST = 1509.33, \quad n = 24$

On the basis of these values, which order model would you choose? Explain your reasoning.

(b) (14) Test the null hypothesis that the highest degree term in your chosen model is not necessary.