Solution to Second Midterm Practice

1. See pp. 153 and 158 of the notes.

2. (b)

3. X_t given X_{t-1}

4. (b)

5. (a) $\log(1-\theta_i)$

(b) $\log\left(\frac{\theta_i}{\mu_i}\right) = \alpha + \beta x_i \implies$

μ-link is $g(\mu_i) = \log\left(\frac{\theta_i}{\mu_i}\right)$

(c) Take $\log(1-\theta_i) = \alpha + \beta x_i$.

$$\ell(\alpha,\beta) = 10\sum_{i=1}^{n} \log\left(1 - e^{\alpha+\beta x_i}\right) + \sum_{i=1}^{n} y_i(\alpha+\beta x_i) + C$$

$$\frac{\partial^2 \ell}{\partial \alpha^2} = -10\sum_{i=1}^{n} e^{\alpha+\beta x_i}\left(1-e^{\alpha+\beta x_i}\right)^{-2}$$

$$\frac{\partial^2 \ell}{\partial \beta^2} = -10\sum_{i=1}^{n} x_i^2 e^{\alpha+\beta x_i}\left(1-e^{\alpha+\beta x_i}\right)^{-2}$$

$$\frac{\partial^2 \ell}{\partial \alpha \partial \beta} = -10 \sum_{i=1}^{n} x_i e^{\alpha+\beta x_i} \left(1 - e^{\alpha+\beta x_i}\right)^{-2}$$

$$I(\alpha, \beta) = -\begin{bmatrix} \frac{\partial^2 \ell}{\partial \alpha^2} & \frac{\partial^2 \ell}{\partial \alpha \partial \beta} \\ \frac{\partial^2 \ell}{\partial \alpha \partial \beta} & \frac{\partial^2 \ell}{\partial \beta^2} \end{bmatrix}$$

6. (a)