Assignment #5

1. Do Exercise 1 in Chapter 16 of GCSR. **Hint:** Expand $L(y_i|\eta_i, \phi)$ in a Taylor series about $\hat{\eta}_i$, substitute $\hat{\phi}$ for ϕ, and then complete the square in order to identify z_i and σ_i^2 in $-(\eta_i - z_i)^2/(2\sigma_i^2)$.

2. Let p_1 and p_2 be *fixed* polynomials of the form

$$p_1(x) = a_0 + a_1 x \quad \text{and} \quad p_2(x) = b_0 + b_1 x + b_2 x^2,$$

where $a_1 \neq 0$ and $b_2 \neq 0$.

(a) For any given quadratic $q(x) = \beta_0 + \beta_1 x + \beta_2 x^2$, show that there exist α_0, α_1 and α_2 such that $q(x) = \alpha_0 + \alpha_1 p_1(x) + \alpha_2 p_2(x)$ for all x. Do so by finding explicit expressions for α_0, α_1 and α_2.

(b) Suppose one wishes to maximize the functional $\mathcal{F}(q)$. This amounts to finding the coefficients β_0, β_1, and β_2 that maximize $\mathcal{F}(q)$. Suppose instead that one chooses α_0, α_1 and α_2 to maximize $\mathcal{F}(\alpha_0 + \alpha_1 p_1(x) + \alpha_2 p_2(x))$. If $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$ and $(\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2)$ are the solutions to the respective problems, argue that $\hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2 \equiv \hat{\alpha}_0 + \hat{\alpha}_1 p_1(x) + \hat{\alpha}_2 p_2(x)$.

3. For the data in the example of Section 3.7, use Newton-Raphson to find the MLEs of model parameters for both logit and probit models. Determine the normal approximation to the posterior distribution in each case assuming that a uniform prior is used for the model parameters.