Proof that asymptotic normality implies consistency

Without loss of generality, we'll assume that $\sigma(\theta) = 1$.

We have

$$P(\left| T_n - \tau(\theta) \right| < \epsilon) =$$

$$P(-\epsilon < T_n - \tau(\theta) < \epsilon) =$$

$$P(T_n - \tau(\theta) < \epsilon) - P(T_n - \tau(\theta) \leq -\epsilon) =$$

$$P(\sqrt{n}(T_n - \tau(\theta)) < \sqrt{n}\epsilon) -$$

$$P(\sqrt{n}(T_n - \tau(\theta)) \leq -\sqrt{n}\epsilon).$$

Need to show that for $1 > \delta > 0$, there exists $n_0$ such that for all $n > n_0$ $P(\left| T_n - \tau(\theta) \right| < \epsilon) > 1 - \delta$.

Let $z > 0$ be such that $\Phi(z) - \Phi(-z) = 2\Phi(z) - 1 = 1 - \delta/2$. 

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For $\sqrt{n}\epsilon > z$, or $n > (z/\epsilon)^2$, 

$$P(\sqrt{n}(T_n - \tau(\theta)) < \sqrt{n}\epsilon) -$$

$$P(\sqrt{n}(T_n - \tau(\theta)) \leq -\sqrt{n}\epsilon) \geq$$

$$P(\sqrt{n}(T_n - \tau(\theta)) \leq z) -$$

$$P(\sqrt{n}(T_n - \tau(\theta)) \leq -z).$$

Because of the asymptotic normality of $\{T_n\}$, the last expression tends to $1 - \delta/2$. So, there exists $n_1$ such that for $n > n_1$

$$P(\sqrt{n}(T_n - \tau(\theta)) \leq z) -$$

$$P(\sqrt{n}(T_n - \tau(\theta)) \leq -z) > 1 - \delta.$$

The proof is finished upon defining 

$$n_0 = \max(n_1, (z/\epsilon)^2).$$