STAT611, Semester II 2007-2008

Assignment 11

(Deadline: 04/21/2008)

1. (Ex10.1) A random sample $X_1, \ldots, X_n$ is drawn from a population with pdf

$$f(x|\theta) = \frac{1}{2}(1 + \theta x), \quad -1 < x < 1, \quad -1 < \theta < 1.$$

   Find a consistent estimator of θ and show that it is consistent.

2. (Ex10.3) A random sample $X_1, \ldots, X_n$ is drawn from a population that is $N(\theta, \theta)$, where $\theta > 0$.

   (a) Show that the MLE of θ, $\hat{\theta}$ is a root of the quadratic equation $\theta^2 + \theta - W = 0$, where $W = (1/n)\sum_{i=1}^n X_i^2$, and determine which root equals the MLE.

   (b) Find the approximate variance of $\hat{\theta}$ using the techniques of Section 10.1.3.

3. (Ex10.9) Suppose that $X_1, \ldots, X_n$ are iid Poisson(λ). Find the best unbiased estimator of

   (a) $e^{-\lambda}$, the probability that $X = 0$.

   (b) $\lambda e^{-\lambda}$, the probability that $X = 1$.

   (c) For the best unbiased estimators of parts (a) and (b), calculate the asymptotic relative efficiency with respect to the MLE. Which estimators do you prefer? Why?

4. (Ex10.34) For testing $H_0 : p = p_0$ versus $H_1 : p \neq p_0$, suppose we observe $X_1, \ldots, X_n$ iid Bernoulli(p).

   (a) Derive an expression for $-2 \log \lambda(x)$, where $\lambda(x)$ is the LRT statistic.

   (b) As in Example 10.3.2, simulate the distribution of $-2 \log \lambda(x)$ and compare it to the χ^2 approximation.

5. (Ex10.35) Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$ population.

   (a) If μ is unknown and σ^2 is known, show that $Z = \sqrt{n}(\bar{X} - \mu_0)/\sigma$ is a Wald statistic for testing $H_0 : \mu = \mu_0$.

   (b) If σ^2 is unknown and μ is known, find a Wald statistic for testing $H_0 : \sigma = \sigma_0$.