STAT414, Semester I 2009-2010

Assignment 9

(Deadline: 11/06/2009)

1. (Ex4.89) Let $X_1, \ldots, X_n$ are independent normal random variables with means μ_i and variances σ_i^2. Show that $Y = \sum_{i=1}^n \alpha_i X_i$, where the α_i are scalars, is normally distributed, and find its mean and variance.

2. (Ex4.101) Find the approximate mean and variance of $Y = \sqrt{X}$, where X is a random variable following a Poisson distribution.

3. Let $X_1, X_2, \ldots$ be a sequence of independent random variables with $E(X_i) = \mu$ and $\text{Var}(X_i) = \sigma_i^2$. Show that if $n^{-2} \sum_{i=1}^n \sigma_i^2 \to 0$, then $\bar{X} \to \mu$ in probability.

4. Let X_i be as in Problem 3 but with $E(X_i) = \mu_i$ and $n^{-1} \sum_{i=1}^n \mu_i = \mu$. Show that $\bar{X} \to \mu$ in probability.

5. A six-sided die is rolled 100 times. Using the normal approximation, find the probability that the face showing a six turns up between 15 and 20 times. Find the probability that the sum of the face values of the 100 trials is less than 300.

6. Suppose that you bet $5 on each of a sequence of 50 independent fair games. Use the central limit theorem to approximate the probability that you will lose more than $75.