

## Assignment 8

(Deadline: 10/30/2009)

1. (4.73) A fair coin is tossed  $n$  times, and the number of heads,  $N$ , is counted. The coin is then counted  $N$  times. Find the expected total number of heads generated by this process.
2. (Ex4.75) Let  $T$  be an exponential random variable, and conditional on  $T$ , let  $U$  be uniform on  $[0, T]$ . Find the unconditional mean and variance of  $U$ .

3. (Ex4.77) Let  $X$  and  $Y$  have the joint density

$$f(x, y) = e^{-y}, \quad 0 \leq x \leq y.$$

- (a) Find  $\text{Cov}(X, Y)$  and the correlation of  $X$  and  $Y$ .
  - (b) Find  $E(X|Y = y)$  and  $E(Y|X = x)$ .
  - (c) Find the density functions of the random variables  $E(X|Y)$  and  $E(Y|X)$ .
4. (Ex4.79) Let  $X$  be a discrete random variable that takes on values 0,1,2 with probabilities  $1/2, 3/8, 1/8$ , respectively. Find the moment-generating function of  $X$ ,  $M(t)$ , and verify that  $E(X) = M'(0)$  and that  $E(X^2) = M''(0)$ .
  5. (Ex4.80) Let  $X$  be a continuous random variable with density function  $f(x) = 2x, 0 \leq x \leq 1$ . Find the moment-generating function of  $X$ ,  $M(t)$ , and verify that  $E(X) = M'(0)$  and that  $E(X^2) = M''(0)$ .
  6. (Ex4.88) Let  $X$  and  $Y$  be independent random variables, and let  $\alpha$  and  $\beta$  be scalars. Find an expression for the mgf of  $Z = \alpha X + \beta Y$  in terms of the mgf's of  $X$  and  $Y$ .