

Assignment 6

(Deadline: 10/16/2009)

1. (EX3.64) Find the joint density of $X + Y$ and X/Y , where X and Y are independent exponential random variables with parameter λ . Show that $X + Y$ and X/Y are independent.
2. (Ex3.79) If T_1 and T_2 are independent exponential random variables, find the density function of $R = T_{(2)} - T_{(1)}$.
3. (Ex3.80) Let U_1, \dots, U_n be independent uniform random variables, and let V be uniform and independent of the U_i .
 - (a) Find $P(V \leq U_{(n)})$.
 - (b) Find $P(U_{(1)} < V < U_{(n)})$.
4. (Ex4.6) Let X be a continuous random variable with probability density function $f(x) = 2x$, $0 \leq x \leq 1$.
 - (a) Find $E(X)$.
 - (b) Let $Y = X^2$. Find the pdf of Y and use it to find $E(Y)$.
 - (c) Use Theorem A in Section 4.1.1 to find $E(X^2)$ and compare to your answer in part (b).
 - (c) Find $\text{Var}(X)$.
5. (Ex4.10) A list of n items is arranged in random order, to find a requested item, they are searched sequentially until the desired item is found. What is the expected number of items that must be searched through, assuming that each item is equally likely to be the one requested?
6. (EX4.13) If X is a nonnegative continuous random variable, show that

$$E(X) = \int_0^{\infty} [1 - F(x)] dx.$$

Apply this result to find the mean of the exponential distribution.