STAT414, Semester I 2009-2010

Assignment 5

(Deadline: 10/09/2009)

1. (EX3.3) Three players play 10 independent rounds of a game, and each player has probability 1/3 of winning each round. Find the joint distribution of the numbers of games won by each of the three players.

2. (EX3.8) Let X and Y have the joint density

$$f(x,y) = \frac{6}{7}(x+y)^2, \quad 0 \le x \le 1, \quad 0 \le y \le 1.$$

   (a) Find (i) $P(X > y)$, (ii) $P(X + Y < 1)$, (iii) $P(X \le 0.5)$.

   (b) Find the marginal density of X and Y.

   (c) Find the two conditional densities.

3. (Ex3.14) Let

$$f(x,y) = xe^{-x(y+1)}, \quad 0 \le x < \infty, \quad 0 \le y < \infty.$$

   (a) Find the marginal densities of X and Y. Are X and Y independent?

   (b) Find the conditional densities of X and Y.

4. (Ex3.19) Suppose that two components have independent exponentially distributed lifetime, T_1 and T_2, with parameters α and β, respectively. Find (a) $P(T_1 > T_2)$ and (b) $P(T_1 > 2T_2)$.

5. (Ex3.37) Let $f(x) = 6x^2(1-x)^2$, for $-1 \le x \le 1$. Describe an algorithm to generate random variables from this density using the rejection method. In what proportion of the trials will be the acceptance step be taken?

6. (Ex3.44) Let N_1 and N_2 be independent random variables following Poisson distributions with parameters λ_1 and λ_2. Show that the distribution of $N = N_1 + N_2$ us Poisson with parameter $\lambda_1 + \lambda_2$.

7. (Ex3.48) Let T_1 and T_2 be independent exponentials with parameters λ_1 and λ_2. Find the density function of $T_1 + T_2$.