STAT414, Semester I 2009-2010

Assignment 4

(Deadline: 10/02/2009)

1. (EX2.33) Let $F(x) = 1 - \exp(-\alpha x^{\beta})$ for $x \geq 0$, $\alpha > 0$, $\beta > 0$, and $F(x) = 0$ for $x < 0$. Show that F is a cdf, and find the corresponding density.

2. (Ex2.38) If f and g are densities, show that $\alpha f + (1-\alpha)g$ is a density, where $0 \leq \alpha \leq 1$.

3. (EX2.40) Suppose that X has the density function $f(x) = cx^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise.

   (a) Find c. (b) Find the cdf. (c) What is $P(0.1 \leq X < 0.5)$?

4. (EX2.53) Let X be a normal random variable with $\mu = 5$ and $\sigma = 10$. Find (a) $P(X > 10)$, (b) $P(-20 < X < 15)$, and (c) the value of x such that $P(X > x) = 0.05$.

5. (EX2.60) Find the density function of $Y = e^Z$, where $Z \sim N(\mu, \sigma^2)$. This is called the lognormal density, since $\log Y$ is normally distributed.

6. (EX2.67) The Weibull cumulative distribution function is

$$F(x) = 1 - e^{-(x/\alpha)^{\beta}}, \quad x \geq 0,\ \alpha > 0,\ \beta > 0.$$

   (a) Find the density function.

   (b) Show that if W follows a Weibull distribution, then $X = (W/\alpha)^{\beta}$ follows an exponential distribution.

   (c) How could Weibull random variables be generated from a uniform random number generator?