STAT414, Semester I 2009-2010

Assignment 3

(Deadline: 09/25/2009)

1. (EX2.1) Suppose that X is a discrete random variable with $P(X = 0) = 0.25$, $P(X = 1) = 0.125$, $P(X = 2) = 0.125$, and $P(X = 3) = 0.5$. Graph the frequency function and the cumulative distribution function of X.

2. (Ex2.15) Two teams, A and B, play a series of games. If team A has probability 0.4 of winning each game, is it to its advantage to play the best three out of five games or the best four out of seven? Assume the outcomes of successive games are independent.

3. (EX2.17) Suppose that in a sequence of independent Bernoulli trials, each with probability of success p, the number of failures up to the first success is counted. What is the frequency function for this random variable?

4. (EX2.17) Continuing with Problem 3, find the frequency function for the number of failures up to the rth success.

5. Let $p_0, p_1, \ldots, p_n$ denote the probability mass function of the binomial distribution with parameters n and p. let $q = 1 - p$. Show that the binomial probabilities can be computed recursively by $p_0 = q^n$ and
$$p_k = \frac{(n-k+1)p}{kq} p_{k-1}, \quad k = 1, 2, \ldots, n.$$
Use this relation to find $P(X \leq 4)$ for $n = 100$ and $p = 0.01$.

6. (EX2.29) Show that the Poisson probabilities p_0, $p_1, \ldots$ can be computed recursively by $p_0 = \exp(-\lambda)$ and
$$p_k = \frac{\lambda}{k} p_{k-1}, \quad k = 1, 2, \ldots.$$
Use this scheme to find $P(X \leq 4)$ for $\lambda = 1$ and compare to the results of Problem 5.

7. (Ex2.30) Suppose that in a city, the number of suicides can be approximately by a Poisson process with $\lambda = 0.33$ per month.

   (a) Find the probability of k suicides in a year for $k = 0, 1, 2, \ldots$ What is the most probable number of suicides?

   (b) What is the probability of two suicides in one week?